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A PRELIMINARY UPPER ATMOSPHERIC DENSITY MODEL DERIVED FROM SATELLITE DRAG DENSITY DATA

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16. ABSTRACT

The object of this study was to develop a preliminary statistical model to better represent the atmospheric density fluctuations in the 170- to 360-km altitude range. A preliminary investigation was carried out to compare satellite drag densities with those calculated under the same conditions using the Jacchia density model. The purpose was to alter the Jacchia model in such a way that the error between model calculation and observed values was minimized. Examination at normalized altitudes of the drag density versus exospheric temperature (from Jacchia's equations) strongly indicated that this parameter could be used to determine density directly, precluding any restrictive assumptions, number density calculations and complicated equations, all used by Jacchia after exospheric temperature is calculated.

Satellite drag-determined densities versus exospheric temperatures were plotted for 10-kilometer altitude increments between 170 and 360 kilometers. Best-fit regression equations for each altitude available were generated. Results of first- through fifth-order fits indicated no appreciable loss in accuracy would result from using a first-order fit. The regression constants were extrapolated down to 100 km to obtain equations for the density at the lower altitudes. The density at 90 km was assumed constant. The resulting linear model was compared with the original Jacchia model in predicting mass density values and the lifetimes of various satellites that have decayed. Results indicate the linear model to be a better mass density predictor than other models considered and at least comparable to the other models with respect to lifetime predictions. The linear model could be further improved by developing a more refined exospheric temperature calculation technique and a more realistic definition of latitudinal density variations.

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FOREWORD

The study described in this report was conducted by personnel of the Dynamic & Guidance Department of Lockheed's Huntsville Research & Engineering Center under contract NAS8-30513 for the Aerospace Environment Division, Aero-Astrodynamics Laboratory, Marshall Space Flight Center,

The objective of the study was to develop a statistical upper atmospheric model from the detailed analysis of satellite drag determined density data. The term "statistical" should be considered in a most general sense and is used only to identify the model as being unrelated to the physical processes of the atmosphere, such as diffusive equilibrium, molecular dissociation, heat absorption, etc.

The overall plan is to compare the statistical model in the other upper atmospheric models, that consider the atmospheric physical processes, which are currently being developed by the Aerospace Environment Division. This comparative approach will provide a control check on each of the models and will generate a more refined upper atmospheric model than could be obtained from any of the individual studies.

The contract under which this study was conducted began November 19, 1968, and ended August 19, 1969.

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SUMMARY

The object of this study was to develop a preliminary statistical model to better represent the atmospheric density fluctuations in the 170- to 360-kilometer altitude range.

A preliminary investigation was carried out to compare satellite drag densities with those calculated under the same conditions using the Jacchia density model. The purpose was to alter the Jacchia model in such a way that the error between model calculation and observed values was minimized. Examination at normalized altitudes of the drag density versus exospheric temperature (from Jacchia's equations) strongly indicated that this parameter could be used to determine density directly, precluding any restrictive assumptions, number densities calculations and complicated equations, all employed by Jacchia after exospheric temperature is calculated.

Satellite drag-determined densities versus exospheric temperatures were plotted for 10-kilometer altitude increments between 170 and 360 kilometers. Best-fit regression equations for each altitude available were generated. Results of first- through fifth-order fits indicated no appreciable loss in accuracy would result from using a first-order fit. The regression constants were extrapolated down to 100 kilometers to obtain equations for the density at the lower altitudes. The density at 90 kilometers was assumed constant. The resulting linear model was compared with the original Jacchia model in predicting mass density values and also in predicting the lifetimes of various satellites that have decayed. The model was also compared with two other existing density models.

Results indicate the linear model to be a better mass density predictor than other models considered and at least comparable to the other models with respect to lifetime predictions.

Results also indicate that the linear model could be further improved by developing a more refined exospheric temperature calculation technique and a more realistic definition of latitudinal density variations.

CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

Section 1
INTRODUCTION

Prior to the initial satellite launchings 12 years ago, it was realized that the tracking of earth satellites would provide a valuable technique for deriving air density in the upper atmosphere from the decay of their orbital period. It was probably not foreseen, however, that satellite launchings would be taking place on such a frequent basis, and with such a variety of orbital characteristics that it would be possible to build up a fairly comprehensive picture of the density structure of the upper atmosphere in the span of time which has elapsed since 1957.

The vast number of satellite and rocket measurements have revealed many different variations in the density of the upper atmosphere. These variations, resulting from the dynamic nature of the atmosphere, have been found to be:

- Diurnal Variation
- Solar Activity Effect (27-day variation)
- Solar Cycle Effect (11-year variation)
- Geomagnetic Activity Effect
- Semi-Annual Variation
- Latitudinal-Seasonal Effect.

Numerous models, derived from density values deduced from satellite drag data, have been developed. Each model has attempted to describe the time-dependent nature of the atmosphere by including one or more of the atmospheric variations. Due to the different assumptions and computational schemes employed by the investigators, their derived values, as well as their models, vary considerably. The more significant attempts will be described in the following paragraphs.

The first multitemperature model that was based on the principle of diffusive equilibrium was given by Nicolet (Ref. 1). The densities were empirically derived from an assumed temperature profile and fixed boundary conditions at 120 km altitude so that they would agree with densities deduced from satellite drag data. This technique provided an acceptable scientific basis upon which more refined models could be developed. Nicolet's model was seriously limited because of three simplifying assumptions: (1) invariant boundary conditions at 120 km; (2) constant temperature gradient between 120 and 150 km; and (3) static equilibrium in an atmosphere that is subject to large day-to-night temperature variations.

Harris and Priester (Ref. 2) accounted for the diurnal variation at low latitudes by simultaneously integrating the hydrostatic equation and heat conduction equation while allowing the heat input to vary with a 24-hour cycle. Since the diurnal variation in the amount of solar radiation necessary to maintain the heat balance was found to be much in excess of that observed, however, a "second heat source" with a maximum at a difference hour was introduced. Although the idea of a second heat source has been questioned, the densities derived are in good agreement with densities derived from satellite drag data. It is limited to low latitudes due to its failure to account for the seasonal migration of the diurnal density bulge. It is also based on constant boundary conditions at 120 km altitude.

Jacchia (Ref. 3), in developing his model, adapted the procedures of Nicolet. Jacchia's boundary conditions are the same as those of the CIRA 1965 Reference Atmosphere except that the helium concentration was increased 40% so the model would agree with drag-derived density values above 600 km altitude.

Starting from constant 120-km boundary conditions and using the diffusion equation and empirically derived temperature profiles, Jacchia calculated number density profiles of each of the atmospheric constituents. The total mass density was then obtained by summing the masses of the constituents. Because of the invariant boundary conditions, non-representative density variations resulted below 200 km altitude.

A low-altitude model was developed by Small (Ref. 4) using densities derived from drag data of low-altitude Air Force-Lockheed satellites. The density computation, unlike Jacchia's, requires yearly mean 10.7 cm solar flux rather than 81-day means. Small's model was developed by fitting constants that define the Harris-Priester curves. The high-altitude limitation of the Harris-Priester model does not induce significant error. In fact, densities of Small's model are in close agreement with those obtained from Jacchia's model.

The 1962 Special Model (Ref. 5), developed jointly by LMSC and MSFC, is an extension of the 1962 U.S. Standard static model to include the effects of solar cycle, geomagnetic activity, and diurnal variations. Corrections to the base standard densities are those used in Small's model. The model can be used up to 700 km. It is limited however as the U.S. Standard itself is, at best, a conservative approximation to the atmosphere.

In developing the Marshall Space Flight Center (MSFC) Static Diffusion Model (1967), which is a computerized version of Jacchia's model, the diffusion equation was integrated by a technique given by Walker (Ref. 6). The temperature dependency of the thermal diffusion factor for hydrogen was obtained from the hydrogen profiles of Jacchia's model.

The MSFC model is simpler and better defined than other existing models; however, the constant boundary limitations do not allow the atmospheric composition and temperature to be realistically defined. This weakness, however, does not limit the accuracy of the mass density defined by the model. The model may be used to obtain a description of the atmpshere from 120 to 1000 km altitude, but like other models it is not completely representative of the atmospheric variations below 200 km altitude.

The purpose of this study was to develop a preliminary statistical atmospheric mass density model using densities deduced from drag data of 58 satellites. In meeting the requirements of the Aerospace Environment Division, the model was to be as free from limiting restrictions as possible, capable of being easily programmed for computer applications while hopefully providing an improved prediction technique.

Succeeding sections will describe the methods used to develop the preliminary density model and comparisons of the newly developed model to other existing density models.

The results are summarized, and recommendations are given for further studies that show promise of improving the model.

Section 2
TECHNICAL DISCUSSION

2.1 MODEL DEVELOPMENT

Two avenues of approach were available for the development of a model to satisfy the specified criteria: (1) constructing an entirely new model from basic principles, and (2) developing an improved model by refining an existing model. The first approach was ruled out for two reasons: a period of performance far in excess of that allotted would be necessary for an effort of this nature, and, more important, to pursue this approach would necessitate ignoring the tremendous amount of work already completed in model development and excluding the wealth of available knowledge. The second approach was therefore selected, and after a brief review of a number of existing models, the Jacchia model was selected because it uses temperature as a basic parameter.

Most of the existing models use density as the basic parameter and relate the dynamic behavior of the atmosphere to density itself. Because the atmospheric parameter most directly affected by solar heating is temperature, it is believed that the most satisfactory results in the development of atmospheric models can be obtained if variations in exospheric temperature can be related to indices of solar and geomagnetic activity. The exospheric temperature computation in Jacchia model proceeds as follows:

The smoothed nighttime minimum temperature, $\bar{T}_o$, can be represented by

$\bar{T}_o = 362 + 3.60\,\bar{F}_{10.7}$ where $\bar{F}_{10.7}$ is the 81-day mean decimetric solar flux

The variation of temperature with one solar rotation is given by

$$T'_o = T_o + 1.8\,(F_{10.7} - \bar{F}_{10.7})$$

where $F_{10.7}$ = daily average of decimetric solar flux

To account for the semi-annual variation

$$T_o = T'_o + \left[0.37 + 0.14\ \sin 2\pi \left(\frac{d - 151}{365}\right)\ F_{10.7}\ \sin 4\pi \left(\frac{d - 59}{365}\right)\right]$$

where d = days counted from 1 January to day in question.

To account for the diurnal variation

$$T = T_o\,(1 + R\,\sin^m\theta)\left(1 + \frac{R\,\cos^m\eta - \sin^m\theta}{1 + R\,\sin^m\theta}\ \cos^n\ \frac{\tau}{2}\right)$$

where

$$\tau = H + \beta + \rho\,\sin(H + \gamma)$$

$$\eta = 1/2\,(\phi - \delta_o)$$

$$\theta = 1/2\,(\phi + \delta_o)$$

and

φ = latitude of perigee of satellite

δ_o = declination of sun

H = hour angle of the sun

and the constants

$$R = \underline{0.28}$$

$$m = n = \underline{2.5}$$

$$\beta = \underline{-45}\ \text{deg}$$

$$p = \underline{12}\ \text{deg}$$

$$\gamma = \underline{+45}\ \text{deg}$$

To account for variations with geomagnetic activity

$$\Delta T = \underline{1.0}\ a_p + \underline{100} \left[1 - \exp(0.08\ a_p)\right]$$

where a_p = geomagnetic index.

Thus, the exospheric temperature T_∞ is given by

$$T_\infty = T + \Delta T$$

The quantities underlined are constants that were empirically determined by Jacchia. Previous studies have shown that the calculated density is relatively insensitive to variations in many of these constants. However, some of the constants are critical in the calculation of the density. Succeeding sections will explore this in more detail. After a value of the exospheric temperature has been computed, the corresponding value of the kinetic temperature can be determined for any altitude. Using this value and Walker's analytic solution to the diffusive equilibrium equation, number densities for the constituents of the atmosphere can be found and from these values density is immediately determined. Results of this method, however, are not in good agreement with satellite drag-determined density data below 200 km altitude.

A preliminary investigation was carried out to compare satellite drag-determined densities with those calculated under identical conditions using Jacchia's model. The objective was to alter the model in such a way that the error between the model density calculations and the drag-determined values was minimized.

The data sample used consisted of about 1900 density data points and associated information i.e., modified Julian date, solar-geophysical data corresponding to the date, and orbital data necessary to fix the perigee point in time and space. The data were taken, as recorded, from Smithsonian Astrophysical Observatory reports and Lockheed Tracking Notes. No attempt was made to determine if the methods of density determination from both sources were identical. It was assumed to be so.

Drag-determined densities and Jacchia's exospheric temperature parameter were found to be highly correlated at normalized altitudes indicating that density might be extracted directly once the exospheric temperature was computed. If such was the case, it would preclude calculations of temperature profiles and number densities. Most important, it would remove the assumption of a specific temperature profile — the assumption of diffusive equilibrium from the 120-km boundary upward, and the invalid assumption that the mean molecular weight of the combination of the constituent gases in constants at any given height.

Satellite drag-determined densities were first normalized to the nearest 10-km level and then plotted versus exospheric temperature (from Jacchia's equations) for each 10 km altitude between 130 and 360 km (range of data) as shown in Figs. 1a through 1g. Best-fit regression equations for each altitude available were generated. First-order fits were selected since the results of first- through fifth-order fits indicated no appreciable difference in accuracy.

The equations were of the form

$$\rho_z = a_o + a_1 \, T_\infty$$

where

ρ_z is the density at given 10 km intervals

$a_o \equiv$ intercept value

$a_1 \equiv$ slope value

$T_\infty \equiv$ exospheric temperature.

The intercept values were replaced by base density values generated by substituting $T_\infty = 900^oK$ into the linear equations. The slope values were normalized by dividing each value by the base density value for that altitude. This was done so that the resulting values would represent the percent change of slope per change in altitude.

The base density (intercept) values and the normalized slope values were extrapolated down to 100 km to obtain equations for the density at altitudes below 130 km.

The density at 90 km was assumed to be constant. The value was taken from the 1962 U.S. Standard Atmosphere. The normalized slope at 90 km was taken to be zero. By normalizing the slope values the transition to zero slope at 90 km was easily effected. With a base density defined and a slope defined, density for any exospheric temperature, for any altitude, could be found. Densities between any 10-km levels were interpolated logarithmically.

A least-squares curve was fit to the normalized slope values. The equation utilized was

$$\text{Slope(Norm.)} \times 10^2 = 0.028700 \times Z + 0.000551 \times Z^2$$

where Z = altitide (km).

Thus, the general equation for the Linear density model was

$$\rho_z = \rho_{(900)_Z} \times \left[1 + \text{Slope(Norm.)} \times (T_\infty - 900)\right]$$

The table of base density values and the curve for the normalized slope values are given in Section 3.

2.2 MODEL EVALUATION

The first approach in the evaluation procedures was to compare the drag-determined densities from each of the 58 satellites in the sample with the densities calculated under identifical conditions using the Linear model. The percent deviation between model calculation and drag-determined values was determined for all points listed for each satellite. The standard deviation of the error for each satellite was determined by summing the squares of the $\Delta\rho$'s for each satellite, dividing by the number of points minus one, and then extracting the square root. The results for each satellite were compared with the sigma values that would result using the Jacchia model. The same type of comparison was made with the 1967 LMSC (Small) model and the 1962 U. S. Special model. On the basis of the results, certain inferences were made, which are given in the next section.

The second approach was to employ the Linear model in the MSFC/LMSC Earth Orbital Decay and Lifetime program to predict the lifetimes of 54 satellites that have decayed. It should be pointed out that many of the satellites used in the lifetime analysis were the same as those used in the

58 satellite density analysis mentioned previously. Also, the ballistic parameter ($C_D A/m$) for each satellite used in the lifetime analysis must be known. Area-to-mass ratios are given in many tracking reports and orbital data reports. The constant value for C_D of 2.2 was adopted for this study in order to preclude any confusion in the interpretation of the data. Results from the prediction of the Linear model were compared with the actual lifetimes. As a relative indicator of the merit of the model, the ratios of the actual lifetimes to the computed lifetimes were used. The statistical mean and standard deviation of the ratios were calculated. Results were compared with those generated using the Jacchia model, and results using the 1962 U. S. Special models and the 1967 LMSC model were tabulated.

2.3 MODEL IMPROVEMENT

The Linear model, as developed during the course of the study can be only as good as the regression equations for the density versus exospheric temperature data. Results indicate that where the Jacchia model and the 1967 LMSC models fell down in lifetime predictions, the Linear model also failed. Since all three are related to the same exospheric temperature, the only improvement seemed to be in refining the temperature prediction technique. This in effect will move the data points nearer the mean regression line as the slope and intercept values are fixed.

In examining the exospheric temperature calculation, note that latitudinal-seasonal variations are not included. As the physical energy source is believed to be different at high latitudes than that at lower latitudes, the latitudinal density variations should also be different. There is, however, very little data available to investigate this latitudinal variation in the density.

The linear fits were generated without regard for possible latitudinal dependency of density. There are strong indications, however, that improvement in the model with respect to lifetime prediction can be achieved by correcting the density (from the Linear model) at the high latitudes. Limited time precluded any detailed analysis for a quantitative solution to this problem.

The Linear model may also be improved by establishing an optimal set of empirical constants for the exospheric temperature equations. These constants were empirically determined by Jacchia from almost exclusive analysis of the high-altitude Explorer satellites. Sensitivity studies indicated that the calculated density is relatively insensitive to variations in most of these constants; however, a few are critical in the calculation of the denstiy. In the calculation of exospheric temperature, a correction is made for the 27-day solar variation and the 11-year variation. If these two are combined, the resulting quantity is given by

$$T' = C_1 + C_2 (F_{10.7} + \bar{F}_{10.7})$$

where, as Jacchia used them, the constants are

$$C_1 = 362 \quad \text{and} \quad C_2 = 1.8$$

To account for variations with geomagnetic activity

$$\Delta T = a_p + C_3 \left[1 - \exp (C_4 \, a_p) \right]$$

where these constants are

$$C_3 = 100 \quad \text{and} \quad C_4 = 0.08$$

Using the data sample available, an error minimization technique was employed on the Linear model to determine the best set of these constants so that the error between the drag-determined density and model density was minimized. The technique was used to extract the best set of the constants for discrete altitudes. A general outline of this procedure is given in the Appendix.

The set of altitude-dependent constants was inserted into the Linear model. The lifetimes of the 54 satellites were redetermined using the model with the altitude-varying constants. The table of the C_1, C_2, C_3 and C_4 constants, along with the results of the lifetime analysis, are given in the next section.

The two procedures described are by no means all that can be done to improve the model. These procedures were listed because they seem more obvious. An abundance of good experimental data, in the final analysis, will determine how accurately any model will represent the upper atmospheric density.

Section 3
RESULTS

Figures 1a through 1g show density versus exospheric temperature plots for the altitude ranges available from the data sample. Also shown are the linear fits to the data. At an altitude of 130 km (Fig. 1a), it is obvious that the data are too sparse to expect the fit to be representative of the conditions at this altitude. The data points are from one satellite only (1962 Beta Sigma). Continuing the inspection of the plots, the fit at 160 km is fairly representable, but at 170 km and 180 km neither the slopes nor the intercepts are well defined due to the considerable variability of the data. However, the plots from 190 to 290 km show fairly good relationships between the data and their linear fits. Results at 300 and 310 km are exclusively from two satellites — 1961 Epsilon and 1961 Lambda I. There is considerable spread in the data, and in fact at 310 the slope was negative. No physical reasoning could justify an inversion at this altitude, so the results at this altitude were ignored in the study. The plots of the data at 350 and 360 km are entirely from the analyses of the Explorer I satellite, however, the correlations were excellent.

Results of the linear fits are tabulated in Table 1, along with the number of data points used to derive the fits for each altitude level available. Examination of the table will show that the slope values (a_1) can be set up in table form and interpolation can be performed between any two altitude levels. The intercept values, however, cannot be handled in that manner due to the numerous sign changes. Thus the original intercept values were replaced by base density values. The base density values were generated by substituting $T_\infty = 900^\circ K$ into the linear equations in Table 1.

Base density values are listed in Table 2, and Fig. 2 shows the curve through the normalized slope values. Also shown is the least-squares equation for the values.

Results of the comparison of percent deviation between the observed (drag) density and model density using the Jacchia model and the Linear model are shown in Table 3. The "new" model is seen to have a smaller sigma value than the Jacchia model 40 out of the 58 cases. For the 58 cases, the average sigma was about 16% for the Linear model and 19% for the Jacchia model.

Table 4 shows the same type of comparison as previously stated; however, the Linear model is compared against the Jacchia model, the LMSC model and the 1962 U. S. Special model. As a relative indicator, each sigma value as calculated by each model for the 58 satellites, was ranked 1, 2, 3 or 4 according to whether the sigma value was the smallest, second smallest, second largest, and largest, respectively. The percentage in each rank for each model was determined. The results are shown in Table 5 below.

Table 5

SIGMA VALUE COMPARISONS OF MODELS

Model	Percent of Time in Rank			
	1	2	3	4
Linear	46.55	20.70	18.95	13.80
Jacchia (66)	12.05	32.75	29.35	25.85
67 LMSC	20.70	29.30	39.65	10.35
1962 Special	20.70	17.25	12.05	50.00

Comparing the Linear model with three other models in use today, it was found that the Linear model's sigma value was smallest in nearly half the cases considered. Also worth noting is the fact that in approximately 70% of the cases the sigma value for the Linear model was either the smallest or second smallest. In addition, its percentage in ranks 3 and 4 (approximately 30%) was smaller when compared with any of the remaining models.

Table 6 shows a comparison of lifetime predictions using the Jacchia model and the Linear model. Actual lifetimes is defined here as the time from first orbit determination to earth impact. The ratios of actual-to-computed lifetimes, using the Jacchia model and the Linear model will be used as indicators of the merits of each model. This method is open to question since long lifetimes and short lifetimes, for example, are weighted equally. A graphical illustration of the ratios of actual-to-computed lifetimes for both models are shown in Figs. 3 and 4. The ratios are plotted against the satellite number as they were presented in Table 6. Also shown are the 10% error bands. Figures 5 and 6 give histograms of the ratios for the Linear model and the Jacchia model, respectively. Also given are the means and standard deviations for both models. Figures 7 through 10 show similar graphical illustrations and histograms for the 1967 LMSC model and the 1962 Special Model.

In evaluating the models, a mean close to 1.0 and a small standard deviation is desired. From the results, it is difficult to say which model has a clear advantage over the other. The Linear model has a mean very close to 1.0 but has a standard deviation above 10%. Jacchia's model has a standard deviation less than the Linear model but its mean is farther from 1.0. The 1962 Special has the smallest standard deviation, but its mean shows that it overpredicts lifetimes. Of the four, the 1967 LMSC model seems to fare worse.

Figure 11 is a graphical illustration of the ratio of the Jacchia Density and Linear Model Density versus Altitude for an Exospheric Temperature of $900^{o}K$. The Linear model values are the same as the Base Density values given previously in Table 2. The Jacchia values were generated assuming $T_{\infty} = 900^{o}K$, and then proceeding with his method of density calculation.

Table 7 lists the set of altitude-dependent constants that were the results of the error minimization scheme previously discussed. These constants were substituted into the model and lifetimes were determined for 39 of the 54 satellites. The results of this preliminary analysis are shown in Table 8 where "corrected Linear model" refers to the Linear model with the altitude-dependent constants. Results of 39 of the 54 satellites indicate that no appreciable improvement is made by using the altitude-dependent constants. Two reasons can be offered to explain the results: (1) in utilizing the Error Minimization scheme, only five iterations were performed. The optimal constants may not have been reached at that cutoff point; and (2) it is highly possible that the constants in the calculation of T_∞ are not explicitly functions of altitude. It is reasonable to conjecture that the constants may be time dependent in some manner. When Jacchia developed his model, the constants used were determined from conditions (solar or otherwise) which persisted at that time. Projecting these values for use at a later time could be erroneous. At this time, however, no functional dependence of the "constants" can be offered.

Section 4
DISCUSSION

4.1 SUMMARY OF RESULTS

The main results deduced from the development and evaluation of the preliminary mass density model may be summarized as follows:

a. The Linear model predicts mass density better than the other models considered. The advantage that the Linear model has over the remaining models, especially over Jacchia's, is not as great as expected or desired.

b. In predicting lifetimes of satellites which have decayed, the Linear model has a mean closer to 1.0 than that of any other model though its standard deviation is second highest.

c. The model can be improved by refining the exospheric temperature calculation method. The Linear model developed by this study may be improved by accounting for latitudinal dependency of density.

e. The Linear model is simple to employ, as accurate as the three existing models compared to it, and can be easily corrected or altered.

4.2 ACCURACY OF RESULTS

In order to properly evaluate the results given, it is appropriate to consider the possible sources of error in the analysis employed.

First, by assuming all the data were of the same quality and were derived from identical procedures introduces some error, though no quantitative measure can be attached to it. In regard to the Smithsonian (SAO) data, all the density data were derived from orbital analyses based on satellite positions obtained from field-reduced photographs with Baker-Nunn cameras.

The Lockheed procedure for obtaining atmospheric densities from satellite tracking data relies upon the Lockheed Closed Form Orbit Determination program to supply the orbit parameters for the satellite. From the subsequent ephemeris and range data residuals, the perigee density history for the vehicle is derived.

Inherent in this error is the error due to possible changes in the effective cross-sectional area (due to tumbling) of the satellite, or the possibility that it may have developed thrust. The ballistic coefficients for the Air Force-Lockheed satellites are average quantities for all orientations of the satellite, and corrected for atmospheric rotation. It is probable that the procedures employed by Smithsonian are similar, if not the same. Implicit in the error in the ballistic coefficient is the possible error due to variability in the drag coefficient. The constant value of 2.2 has been adopted by many institutions, though a constant value is known to be a false assertion.

Due to the nature of this analysis there was no way to control the errors mentioned above or others, not mentioned, which were associated with the data sample. But errors from the data sample propagate themselves into the Linear density model, since it was based primarily on the data. It is this point that should be borne in mind.

Secondly, the linear relationship between drag density and exospheric temperature was assumed to be good. The range of exospheric temperatures was from $700^{o}K$ to $1800^{o}K$. The first-order fits were, as a whole, quite good. Exospheric temperatures below about $600^{o}K$ and above $2000^{o}K$ are rare so that the error induced by extending the model to cover these ranges should not be significant.

Also, in initially developing the model, it was assumed in calculating the exospheric temperature (as Jacchia's does it) that the atmospheric variations were accounted for correctly. The Error Minimization technique was used in an attempt to alleviate some of the error associated with this assumption.

Section 5
CONCLUSION

Although the results will not substantiate an assertion that the Linear model is a better model overall than any of those compared to it, it can be stated that the Linear model is at least comparable to either of them. Of more significance, however, is the fact that this preliminary analysis has shown that, based purely on drag density data, a simple statistical mass density model can be developed that can be used to make reliable predictions of the atmospheric density.

Establishing a good density model depends on good experimental data of such quality that atmospheric variations can be clearly discernible. It is recommended that all available low-altitude satellite density data be obtained and used in future studies. The changes in density resulting from geomagnetic storms can be investigated and results used to correct the expression for this variation in the Linear model. Also since some of these satellites were in polar orbits, latitudinal variations of density can be determined in a more quantitative manner. The latitudinal variations would aid in correcting the Linear model for these effects.

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TABLES

Table 1

DENSITY VERSUS EXOSPHERIC TEMPERATURE LINEAR FITS

$$\rho \times 10^{13} = a_o + a_1 (T_\infty/100)$$

Altitude (km)	a_o	a_1	Number of Points
130	-255.64	37.8840	29
160	6.3245	1.0278	78
170	8.6832	0.0344	23
180	4.8262	0.1033	82
190	1.9066	0.2274	312
200	0.3169	0.2317	157
210	-0.1386	0.2188	198
220	0.0461	0.1540	69
230	-0.4326	0.1612	45
240	-0.3737	0.1271	52
250	-0.2140	0.0951	66
260	-0.5094	0.1023	193
270	-0.3972	0.0783	70
280	-0.4393	0.0772	62
290	-0.3769	0.0669	31
300	-0.2135	0.0418	26
310*	0.1758	-0.0031	11
350	-0.1487	0.0248	162
360	-0.1864	0.0258	240
			1886

*Negative slope - results ignored

Table 2

BASE DENSITY (EXOSPHERIC TEMPERATURE = $900^{o}K$)

Altitude (km)	Density ($\times 10^{13}$ gm/cm^3)
90	3.1750×10^{4} *
100	4.9148×10^{3} **
110	9.800×10^{2} **
120	2.5724×10^{2} **
130	85.3160
140	37.7130
150	21.3280**
160	15.5748
170	8.9924
180	5.7554
190	3.9535
200	2.4022
210	1.8308
220	1.4318
230	1.0181
240	0.7704
250	0.6416
260	0.4111
270	0.3072
280	0.2558
290	0.2251
300	0.1657
310	0.1455
320	0.1200**
330	0.1020**
340	0.0950**
350	0.0745
360	0.0461

*Fixed Boundary Value
**Logarithmically Interpolated Values

Table 3

COMPARISON OF PERCENT DEVIATION BETWEEN OBSERVED DENSITY AND MODEL DENSITY

$$\sigma = \sqrt{\frac{\left(\frac{\text{Model - Observed}}{\text{Observed}} \times 100\right)^2}{\text{Number of Points - 1}}}$$

Number	Satellite	Perigee Altitude (Km)	σ - New Model	σ - Jacchia
1	1964 - 32A-2	190	7.047	10.383
2	1964 - 37A-2	190	6.997	8.788
3	1964 - 32A-1	188	6.576	9.514
4	1964 - 75A-1	192	13.737	16.201
5	1964 - 75A-2	190	8.832	18.355
6	1958 Delta 2	194-210	9.920	17.480
7	58 Delta 2	205-227	11.862	10.596
8	59 Gamma	192-256	24.633	22.192
9	59 Epsilon	176-219	26.301	22.195
10	59 Epsilon 2	170-238	12.287	16.520
11	59 Zeta	196-223	17.025	18.194
12	59 Lambda	172-196	9.264	6.416
13	60 Delta	164-173	18.826	3.267
14	60 Theta	207-264	10.890	24.547
15	60 Omicron	159-189	17.974	13.234
16	60 Sigma	207-262	12.292	11.671
17	60 Tau	177-197	5.396	7.532
18	61 Alpha Beta	226-241	15.856	26.415
19	61 Alpha Gamma	197-238	17.001	18.994
20	62 Alpha Epsilon	197-269	21.103	18.890
21	62 Alpha Kappa	197-251	24.360	19.929
22	61 Epsilon	209-315	22.877	23.607
23	61 Zeta	207-271	18.139	19.695
24	61 Lambda 1	246-315	28.570	30.426
25	61 Lambda 2	181-234	12.706	13.101
26	61 Xi	205-225	9.469	13.703
27	61 Pi	194-249	21.892	27.248
28	62 Alpha Gamma	185-215	11.661	15.317
29	62 Alpha Eta	200-204	24.913	45.727
30	62 Alpha Theta	187-208	19.530	23.683
31	62 Alpha Kappa	196-210	7.769	11.842
32	62 Alpha Sigma	166-175	43.238	32.111
33	62 Alpha Chi	194-213	10.093	10.069
34	62 Beta Epsilon	211-217	12.007	18.511
35	62 Beta Omicron	200-210	6.137	13.267
36	62 Beta Sigma	124-135	13.546	11.653
37	62 Beta Phi	187-199	18.442	15.632
38	62 Rho	174-199	23.160	24.282
39	62 Chi	189-215	24.607	31.894
40	63-19A	196-200	10.037	24.441
41	63-25A	205-208	34.00	46.402
42	63-29A	205-210	75.776	87.638
43	63-34A	181-186	30.699	29.317
44	63-55A	184	43.039	38.544
45	63-16A	162	5.748	17.911
46	64-84A	199-213	10.307	14.130
47	Explorer I	354-366	15.831	13.029
48	1963-29A	210	17.615	16.708
49	63-32A	172-173	8.104	11.135
50	63-37A	187-188	8.806	7.622
51	64-37A-1	188-190	5.744	7.159
52	63-48A	184	5.953	5.996
53	63-55A	187	9.947	12.537
54	64-22A-1	189	10.366	11.121
55	64-22A-2	183	14.890	17.137
56	64-27A-1	164	3.163	8.780
57	64-27A-2	188	10.901	19.559
58	63-9A	255-282	19.300	21.711

Table 4

COMPARISON OF PERCENT DEVIATION BETWEEN OBSERVED DENSITY AND MODEL DENSITY

$$\sigma = \sqrt{\frac{\left(\frac{\text{MODEL - OBSERVED}}{\text{OBSERVED}} \times 100\right)^2}{n-1}}$$

No.	Satellite	Altitude	σ Jacchia	Rank	σ New Model	Rank	σ 62 Special	Rank	σ 67 LMSC	Rank
1	1964 32A-2	190	10.383	2	7.047	1	31.238	4	21.775	3
2	1964 37A-2	190	8.788	2	6.997	1	28.934	4	19.312	3
3	1964 32A-1	188	9.514	2	6.576	1	28.580	4	17.411	3
4	1964 75A-1	192	16.201	2	13.737	1	34.267	4	23.183	3
5	1964 75A-2	190	18.355	2	8.832	1	37.351	4	25.352	3
6	1958 Delta 2	194-210	17.480	2	9.920	1	15.490	3	19.162	4
7	58 Delta 2	205-227	10.596	1	11.862	2	14.388	4	12.609	3
8	59 Gamma	192-256	22.192	2	24.633	4	21.734	1	23.518	3
9	59 Epsilon	176-219	22.195	2	26.301	4	19.872	1	23.255	3
10	59 Epsilon 2	170-238	16.520	3	12.287	1	16.572	4	15.740	2
11	59 Zeta	196-223	18.194	3	17.025	2	16.537	1	18.745	4
12	59 Lambda	172-196	6.416	1	9.264	3	10.195	4	8.782	2
13	60 Delta	164-173	3.267	1	18.826	3	25.789	4	16.261	2
14	60 Theta	207-264	24.547	4	10.890	1	16.154	2	20.243	3
15	60 Omicron	159-189	13.234	2	17.974	4	16.471	3	6.787	1
16	60 Sigma	207-262	11.671	2	12.292	3	12.684	4	11.174	1
17	60 Tau	177-197	7.532	2	5.396	1	19.146	4	7.741	3
18	61 Alpha Beta	226-241	26.415	4	15.856	1	21.305	3	21.079	2
19	61 Alpha Gamma	197-238	18.994	4	17.001	2	17.198	3	16.885	1
20	62 Alpha Epsilon	197-269	18.890	3	21.103	4	17.488	2	17.138	1
21	62 Alpha Kappa	197-251	19.929	3	24.360	4	12.202	1	14.382	2
22	61 Epsilon	209-315	23.607	3	22.877	1	23.497	2	24.155	4
23	61 Zeta	207-271	19.695	2	18.139	1	21.910	4	20.700	3
24	61 Lambda 1	246-315	30.426	4	28.570	2	25.373	1	30.017	3
25	61 Lambda 2	181-234	13.101	3	12.706	1	17.993	4	13.030	2
26	61 Xi	205-225	13.703	4	9.469	1	13.369	3	10.871	2
27	61 Pi	194-249	27.248	4	21.892	3	19.279	2	19.134	1
28	62 Alpha Gamma	185-215	15.317	4	11.661	1	13.605	3	11.868	2
29	62 Alpha Eta	200-204	45.727	4	24.913	3	14.356	1	21.617	2
30	62 Alpha Theta	187-208	23.683	4	19.530	3	15.861	2	14.087	1
31	62 Alpha Kappa	196-210	11.842	3	7.769	1	16.641	4	10.328	2
32	62 Alpha Sigma	166-175	32.111	3	43.238	4	24.193	2	10.603	1
33	62 Alpha Chi	194-213	10.069	1	10.093	2	15.050	4	11.149	3
34	62 Beta Epsilon	211-217	18.511	4	12.007	1	13.855	2	18.144	3
35	62 Beta Omicron	200-210	13.267	3	6.137	1	17.527	4	10.243	2
36	62 Beta Sigma	124-135	11.653	2	13.546	3	21.087	4	6.124	1
37	62 Beta Phi	187-199	15.632	2	18.442	3	21.532	4	9.508	1
38	62 Rho	174-199	24.282	3	23.160	2	11.164	1	29.913	4
39	62 Chi	189-215	31.894	4	24.607	3	10.274	1	17.214	2
40	63-19A	196-200	24.441	4	10.037	1	10.418	2	10.703	3
41	63-25A	205-208	46.402	4	34.000	3	33.740	2	32.578	1
42	63-29A	205-210	87.638	4	75.776	3	66.292	2	65.692	1
43	63-34A	181-186	29.317	3	30.699	4	9.989	1	24.563	2
44	63-55A	184	38.544	3	43.039	4	9.960	1	20.490	2
45	1964-84A San Marco	199-213	14.130	4	10.307	2	8.456	1	10.604	3
46	1963-16A	161-163	17.911	2	5.748	1	50.293	4	26.424	3
47	Explorer I	354-366	13.029	1	15.831	2	27.020	4	21.849	3
48	1963-29A	210	16.708	1	17.615	2	22.254	3	23.680	4
49	1963-32A	172-173	11.135	3	8.104	1	39.999	4	9.467	2
50	1963-37A	187-188	7.622	1	8.806	2	30.746	4	18.322	3
51	1964-37A-1	188-190	7.159	2	5.744	1	27.383	4	15.035	3
52	1963-48A	184	5.996	2	5.953	1	33.144	4	15.890	3
53	1963-55A	187	12.537	2	9.947	1	38.100	4	23.748	3
54	1964-22A-1	189	11.121	2	10.366	1	30.884	4	18.114	3
55	1964-22A-2	183	17.137	3	14.890	1	32.589	4	16.001	2
56	1964-27A-1	164	8.780	3	3.163	2	43.846	4	2.792	1
57	1964-27A-2	188	19.559	3	10.901	1	45.550	4	13.137	2
58	1963-9A Explorer 17	255-282	21.711	3	19.300	2	19.058	1	23.597	4

Table 6

COMPARISON OF LIFETIME PREDICTIONS FOR SELECTED SATELLITES

No.	Designation	Actual Lifetime (Days)	Computed Lifetime (Jacchia Model)	Computed Lifetime (Linear Model)	Actual / (Computed Jacchia)	Actual / (Computed Model)
1	58 Delta 2	404.1	360.0	378.63	1.12	1.06
2	58 Delta	197.7	176.0	181.75	1.12	1.08
3	58 Zeta	33.6	30.56	26.97	1.09	1.24
4	59 Gamma	11.2	11.67	12.55	0.96	0.89
5	59 Epsilon	43.4	37.72	39.15	1.15	1.10
6	59 Zeta	60.7	53.86	60.0	1.12	1.01
7	59 Lambda	108.3	100.67	106.34	1.07	1.02
8	59 Epsilon 2	362.	331.42	373.85	1.09	0.97
9	60 Delta	9.83	10.19	9.89	0.96	0.99
10	60 Theta	95.0	94.85	104.2	1.00	0.91
11	60 Omicron	42.9	38.93	40.49	1.10	1.05
12	60 Sigma	107.4	118.0	126.66	0.91	0.85
13	60 Tau	32.9	31.8	34.48	1.03	0.95
14	61 Epsilon	525.5	582.6	614.37	0.90	0.85
15	61 Zeta	422.6	458.39	475.85	0.92	0.89
16	61 Lambda	372.9	381.68	370.72	0.97	1.00
17	61 Lambda	391.2	394.9	417.24	0.99	0.93
18	61 Xi	23.2	21.79	23.02	1.06	1.00
19	61 Pi	133.9	132.0	137.22	1.01	0.97

(Continued)

Table 6 (Continued)

No.	Designation	Actual Lifetime (Days)	Computed Lifetime (Jacchia Model)	Computed Lifetime (Linear Model)	$\frac{\text{Actual}}{\text{(Computed Jacchia)}}$	$\frac{\text{Actual}}{\text{(Computed Model)}}$
20	61 Alpha-Beta	27.3	27.29	28.15	1.00	0.97
21	61 Alpha-Gamma	24.9	25.0	25.26	0.99	0.98
22	61 Alpha-Epsilon	394.3	405.7	406.37	0.97	0.97
23	61 Alpha-Kappa	76.8	66.9	67.33	1.14	1.14
24	62 Rho	15.6	12.0	12.35	1.30	1.26
25	62 Chi	20.6	16.12	16.42	1.27	1.25
26	62 Alpha-Gamma	76.2	75.87	75.30	1.00	1.01
27	62 Sigma	492.0	514.8	517.5	0.95	0.95
28	62 Alpha-Eta	16.5	14.0	14.49	1.17	1.13
29	62 Alpha-Theta	18.6	16.5	17.22	1.12	1.08
30	62 Alpha-Kappa	18.3	18.26	19.24	1.00	0.95
31	62 Alpha-Sigma	6.9	5.59	5.66	1.23	1.21
32	62 Alpha-Chi	56.9	57.6	63.56	0.98	0.90
33	62 Beta-Epsilon	29.5	28.5	30.2	1.03	0.97
34	62 Beta-Omicron	20.1	21.3	22.06	0.94	0.91
35	62 Beta-Sigma	3.6	2.86	2.92	1.25	1.23
36	62 Beta-Phi	16.1	14.89	14.46	1.08	1.11
37	64 Gemini	4.2	3.27	3.11	1.28	1.35
38	64 (SA-6)	3.2	3.23	3.04	0.99	1.05

(Continued)

Table 6 (Continued)

No.	Designation	Actual Lifetime (Days)	Computed Lifetime (Jacchia Model)	Computed Lifetime (Linear Model)	Actual / (Computed Jacchia)	Actual / (Computed Model)
39	64 (SA-7)	3.8	3.06	2.86	1.23	1.33
40	64 (SA-5)	821.4	866.3	874.9	0.94	0.94
41	FTV 1628	11.88	11.62	12.32	1.02	0.96
42	FTV 1626	12.25	13.14	14.03	0.93	0.87
43	FTV 1627	7.5	9.39	9.56	0.79	0.78
44	FTV 1622	9.88	13.39	13.44	0.73	0.73
45	FTV 1623	14.63	16.14	15.89	0.90	0.92
46	65-110A	17.57	17.0	16.82	1.03	1.04
47	64-75A	17.56	16.23	15.55	1.08	1.12
48	64-71A	25.31	25.31	24.59	1.00	1.03
49	64-67A	17.31	22.26	21.65	0.77	0.80
50	64-61A	20.5	24.59	23.57	0.83	0.86
51	64-56A	21.69	22.5	21.85	0.96	0.99
52	64-37A	26.25	25.35	24.04	1.03	1.09
53	64-27A	13.94	14.25	13.66	0.97	1.02
54	63-35A	70.0	63.0	117.25	0.90	0.60

Table 7

PRELIMINARY RESULTS OF ERROR MINIMIZATION SCHEME

Altitude (km)	C1	C2	C3	C4
130	586.0	0.6	94.9	.074
160	396.0	1.6	103.5	.076
170	467.0	1.6	66.2	.017
180	387.6	1.7	123.9	.075
190	367.4	1.8	104.6	.091
200	360.0	1.8	101.0	.087
210	364.9	1.8	98.9	.099
220	344.0	1.8	122.0	.100
230	334.0	1.8	115.0	.137
240	337.1	1.8	105.0	.120
250	368.0	1.8	67.7	.092
260	358.0	1.8	97.7	.089
270	349.7	1.8	110.9	.094
280	351.0	1.9	88.2	.093
290	365.0	1.7	102.2	.103
300	338.0	1.8	104.0	.125
310	Negative Slope — Results Ignored			
350	377.9	1.7	110.0	.110
360	398.4	1.7	98.8	.101

Table 8

COMPARISON OF LIFETIME PREDICTIONS FOR THE LINEAR MODEL AND THE CORRECTED LINEAR MODEL

No.	Designation	Actual Lifetime (Days)	Computed Lifetime (Linear Model)	Computed Lifetime (Corrected Linear Model)
1	58 Delta 2	404.1	378.63	379.29
2	58 Delta	197.7	181.75	182.2
3	58 Zeta	33.6	26.97	27.29
4	59 Gamma	11.2	12.55	12.64
5	59 Epsilon	43.4	39.15	39.32
6	59 Zeta	60.7	60.0	60.52
7	59 Lambda	108.3	106.34	106.33
8	59 Epsilon 2	362.0	373.85	378.15
9	60 Delta	9.83	9.89	9.94
10	60 Theta	95.0	104.2	104.87
11	60 Omicron	42.9	40.49	40.44
12	60 Sigma	107.4	26.66	129.59
13	60 Tau	32.9	34.48	34.47
14	61 Epsilon	525.5	614.37	602.90
15	61 Zeta	422.6	475.85	481.0
16	61 Lambda 1	372.9	370.72	385.5
17	61 Lambda 2	391.2	417.24	417.53
18	61 Xi	23.2	23.02	23.11
19	61 Pi	133.9	137.22	138.2
20	61 Alpha Beta	27.3	28.15	28.50
21	61 Alpha Gamma	24.9	25.26	25.86
22	61 Alpha Epsilon	394.3	406.37	411.11
23	61 Alpha Kappa	76.8	67.33	68.65
24	62 Rho	15.6	12.35	12.33
25	62 Chi	20.6	16.42	16.46
26	62 Alpha Gamma	76.2	75.30	75.31
27	62 Sigma	492.0	517.5	534.14
28	62 Alpha Eta	16.5	14.49	13.99
29	62 Alpha Theta	18.6	17.22	16.75
30	62 Alpha Kappa	18.3	19.24	18.68
31	62 Alpha Sigma	6.9	5.66	5.51
32	62 Alpha Chi	56.9	63.56	61.04
33	62 Beta Epsilon	29.5	30.2	28.75
34	62 Beta Omicron	20.1	22.06	21.33
35	62 Beta Sigma	3.6	2.92	2.92
36	62 Beta Phi	16.1	14.46	13.98
37	64 Gemini	4.2	3.11	3.10
38	64 (SA-6)	3.2	3.04	3.00
39	64 (SA-7)	3.8	2.86	2.82

FIGURES

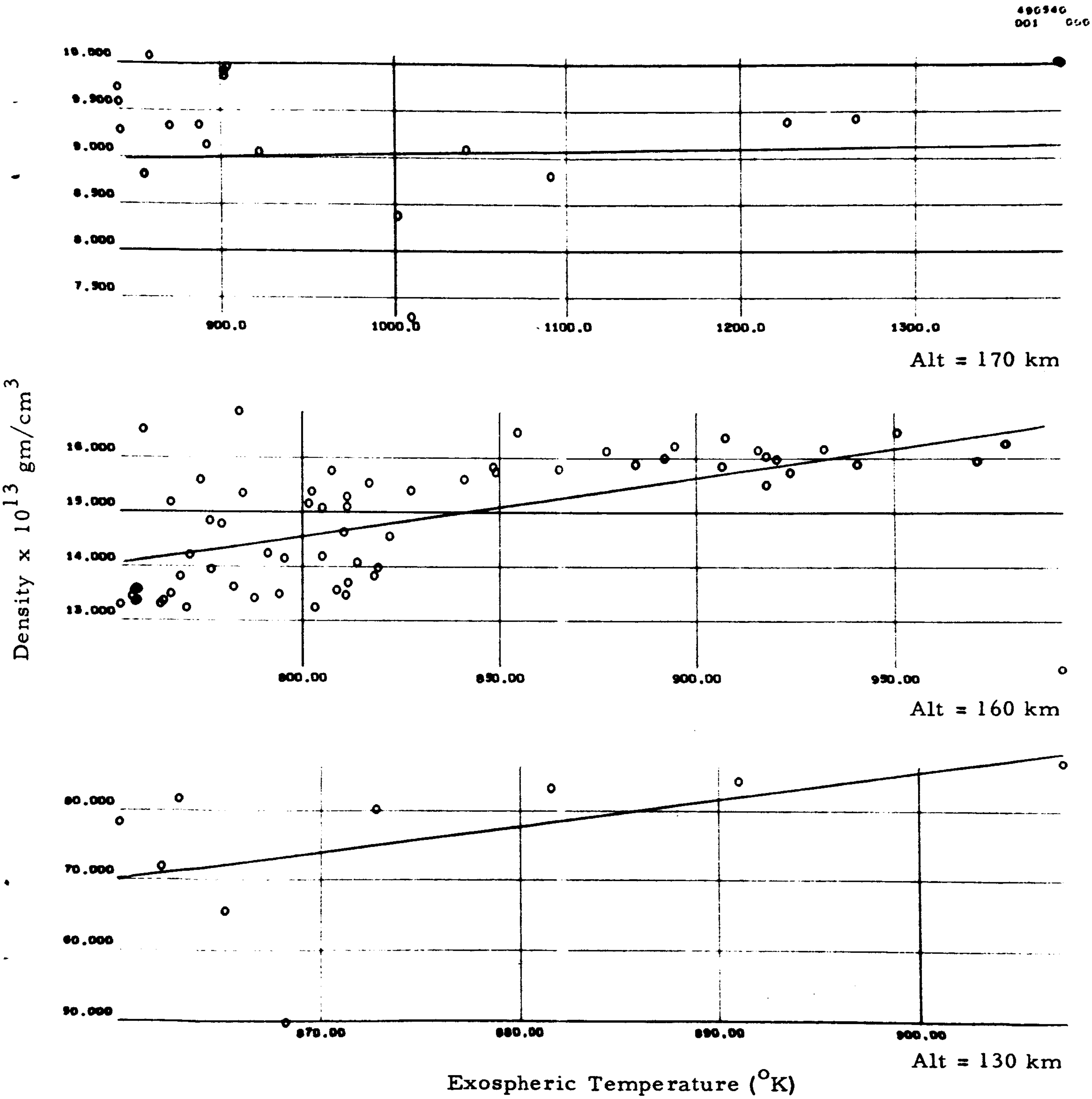

Fig. 1a - Density vs Exospheric Temperature

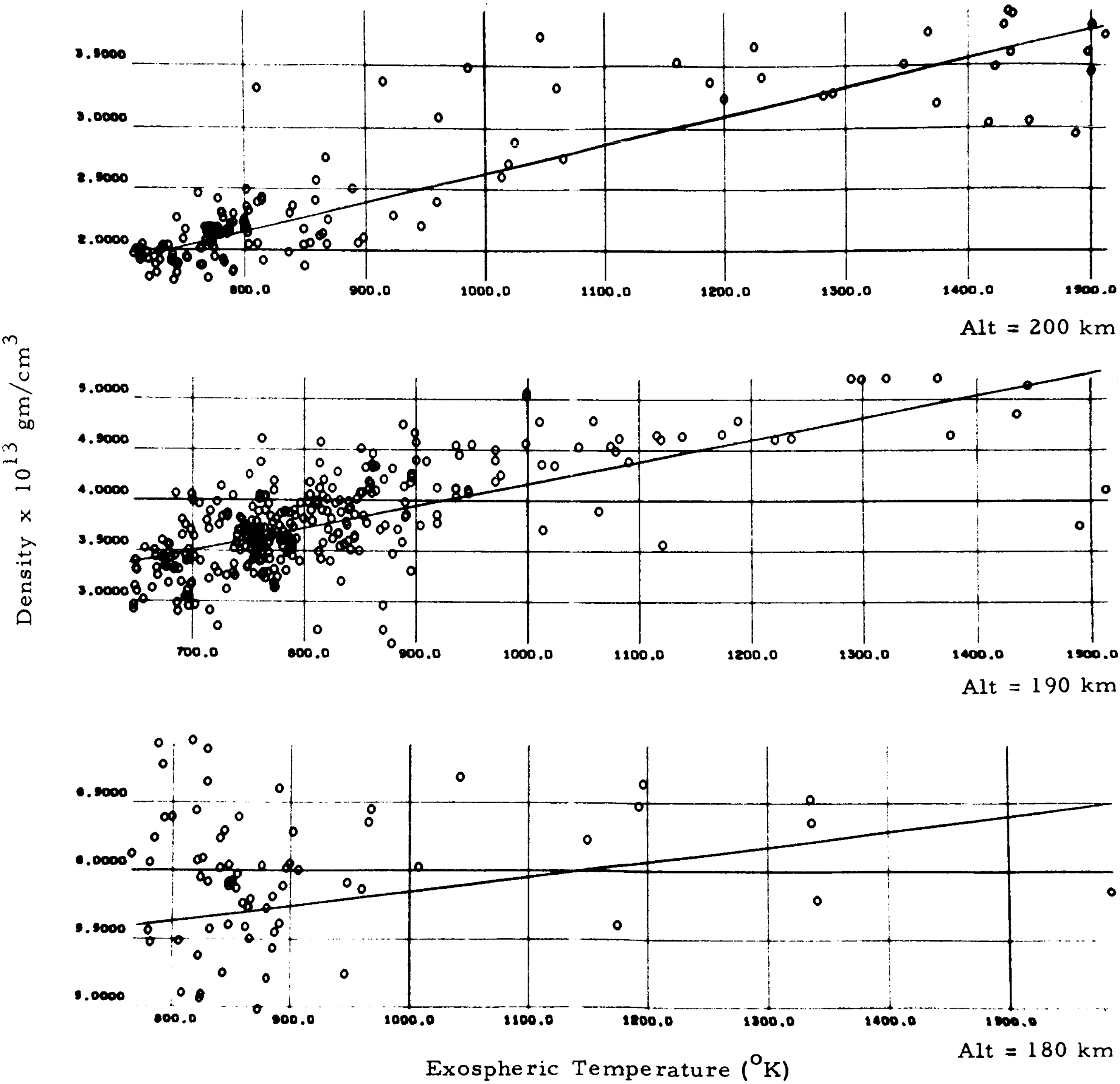

Fig. 1b - Density vs Exospheric Temperature

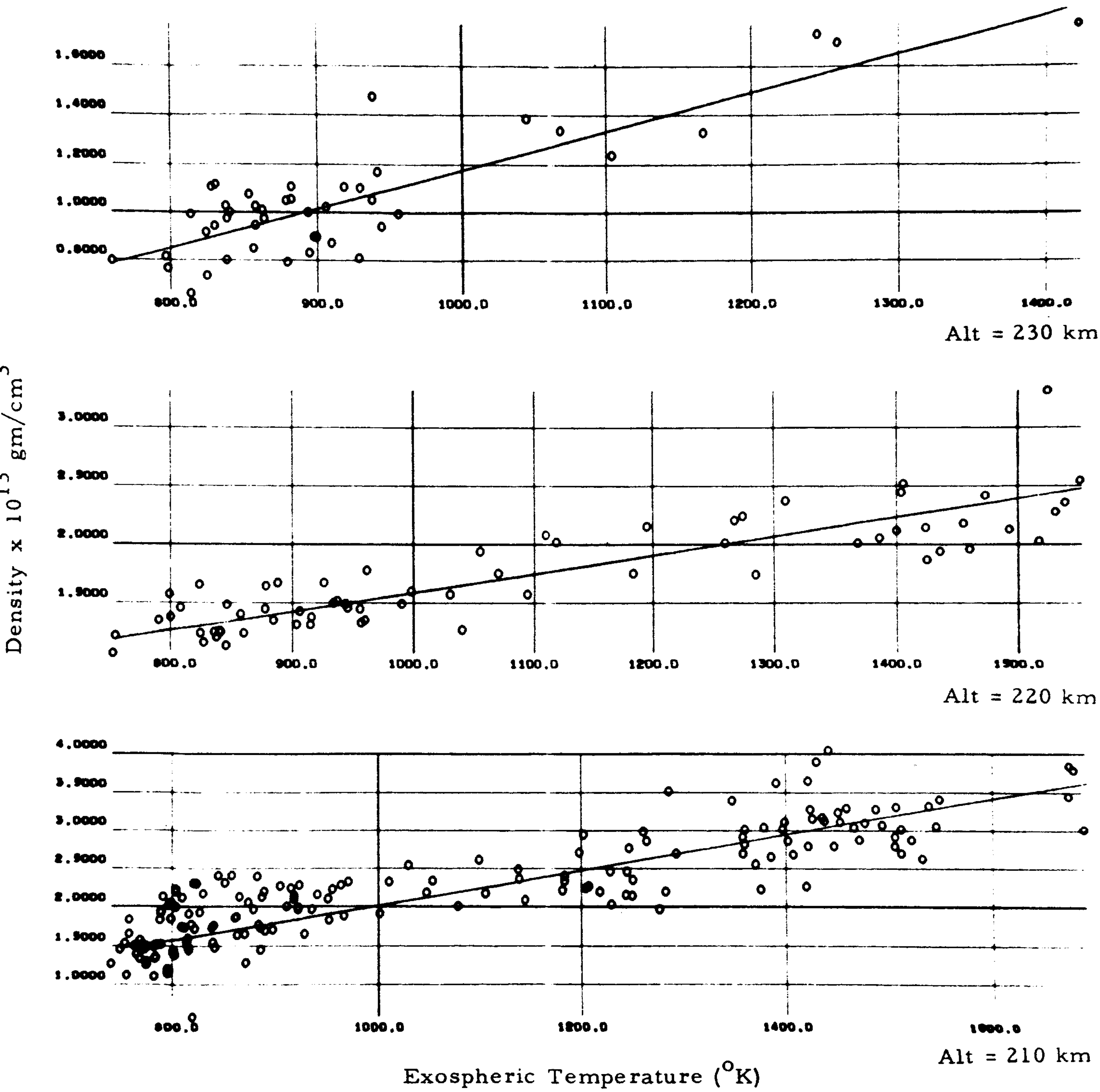

Fig. 1c - Density vs Exospheric Temperature

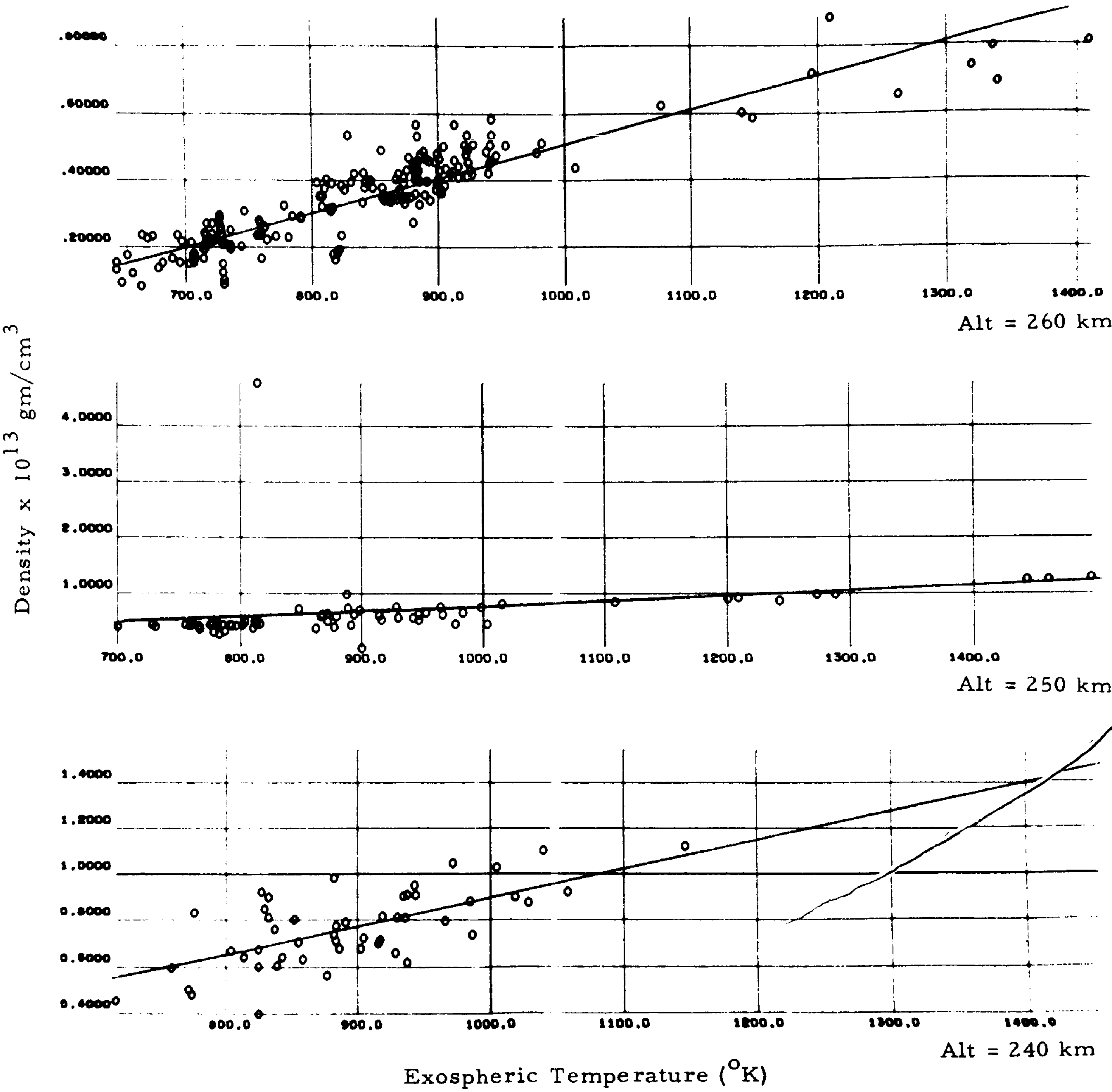

Fig. 1d - Density vs Exospheric Temperature

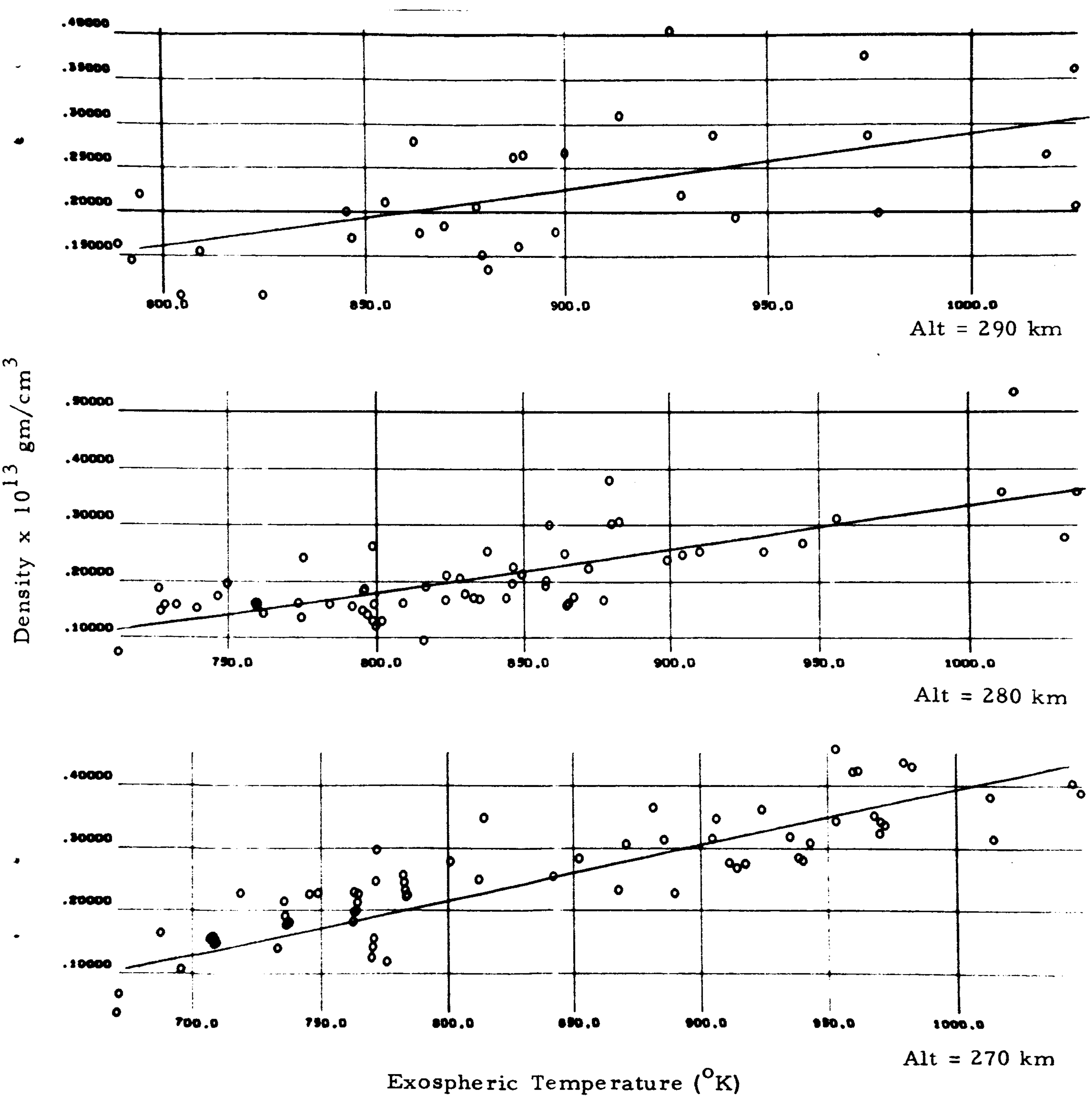

Fig. 1e - Density vs Exospheric Temperature

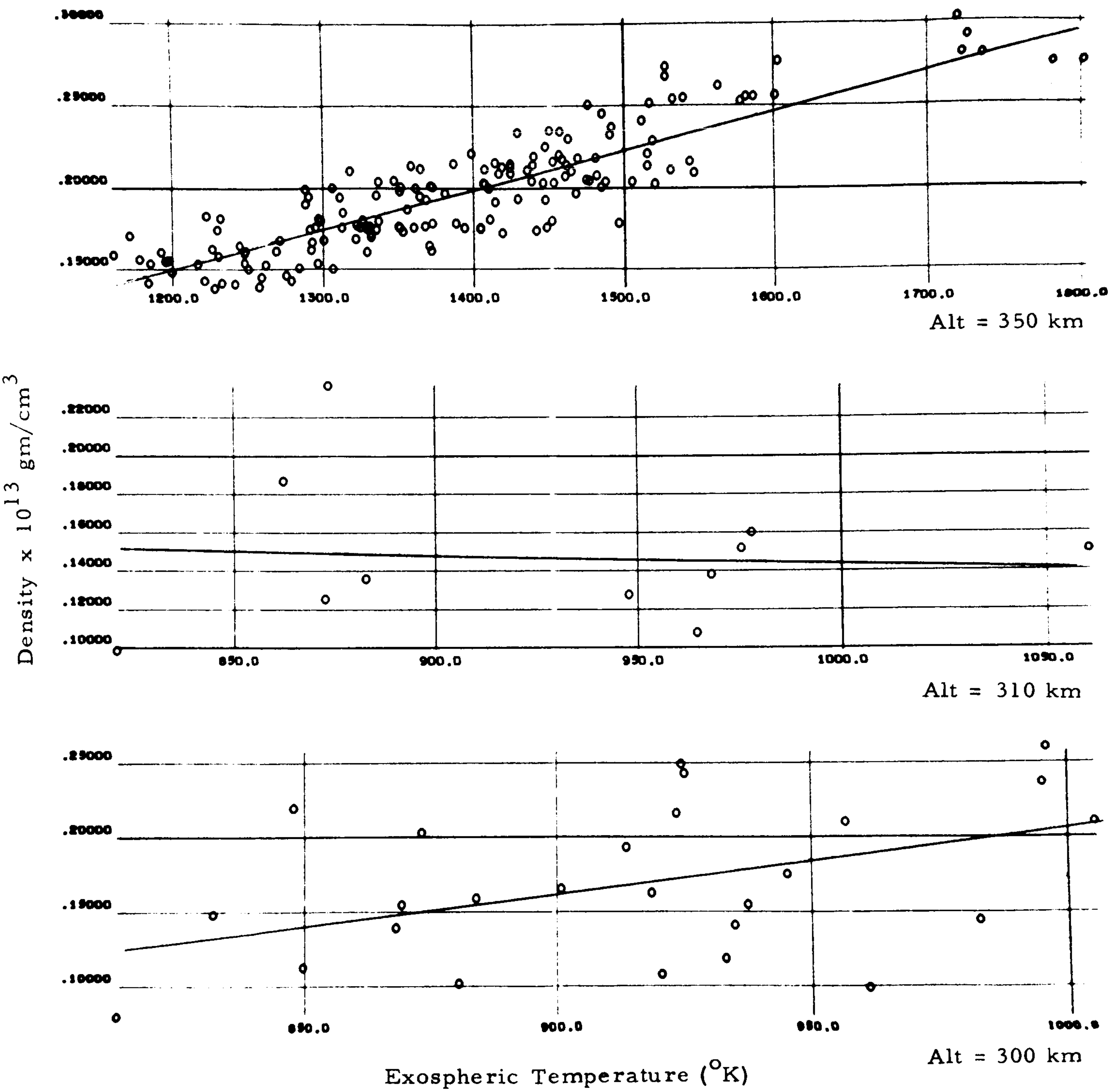

Fig. 1f - Density vs Exospheric Temperature

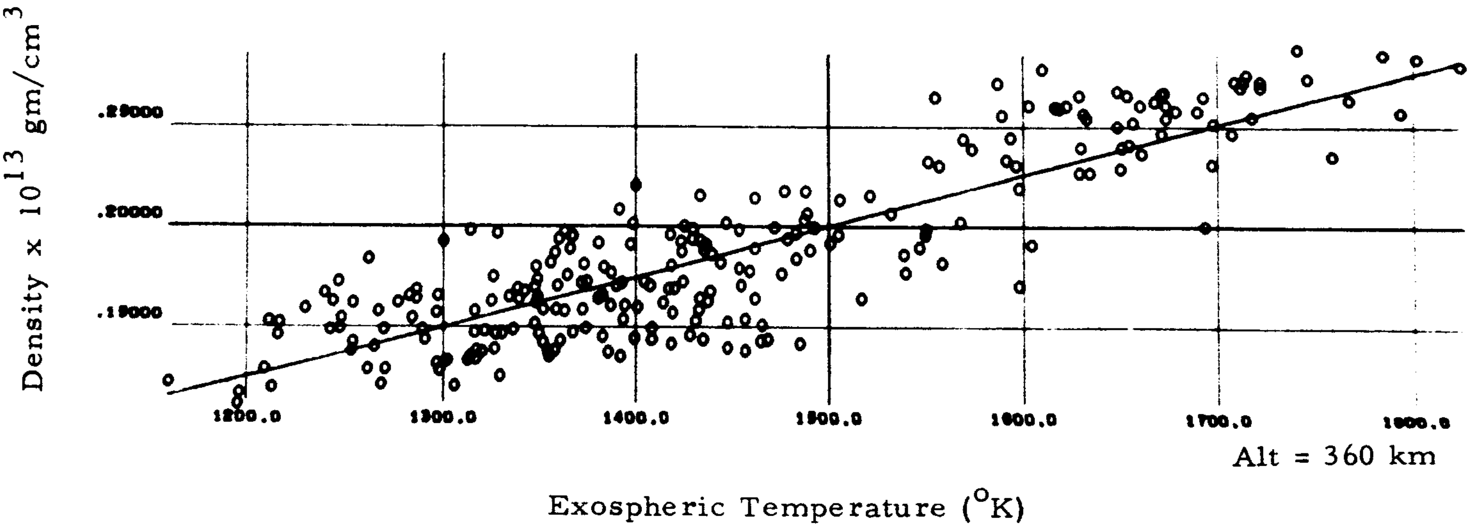

Fig. 1g - Density vs Exospheric Temperature

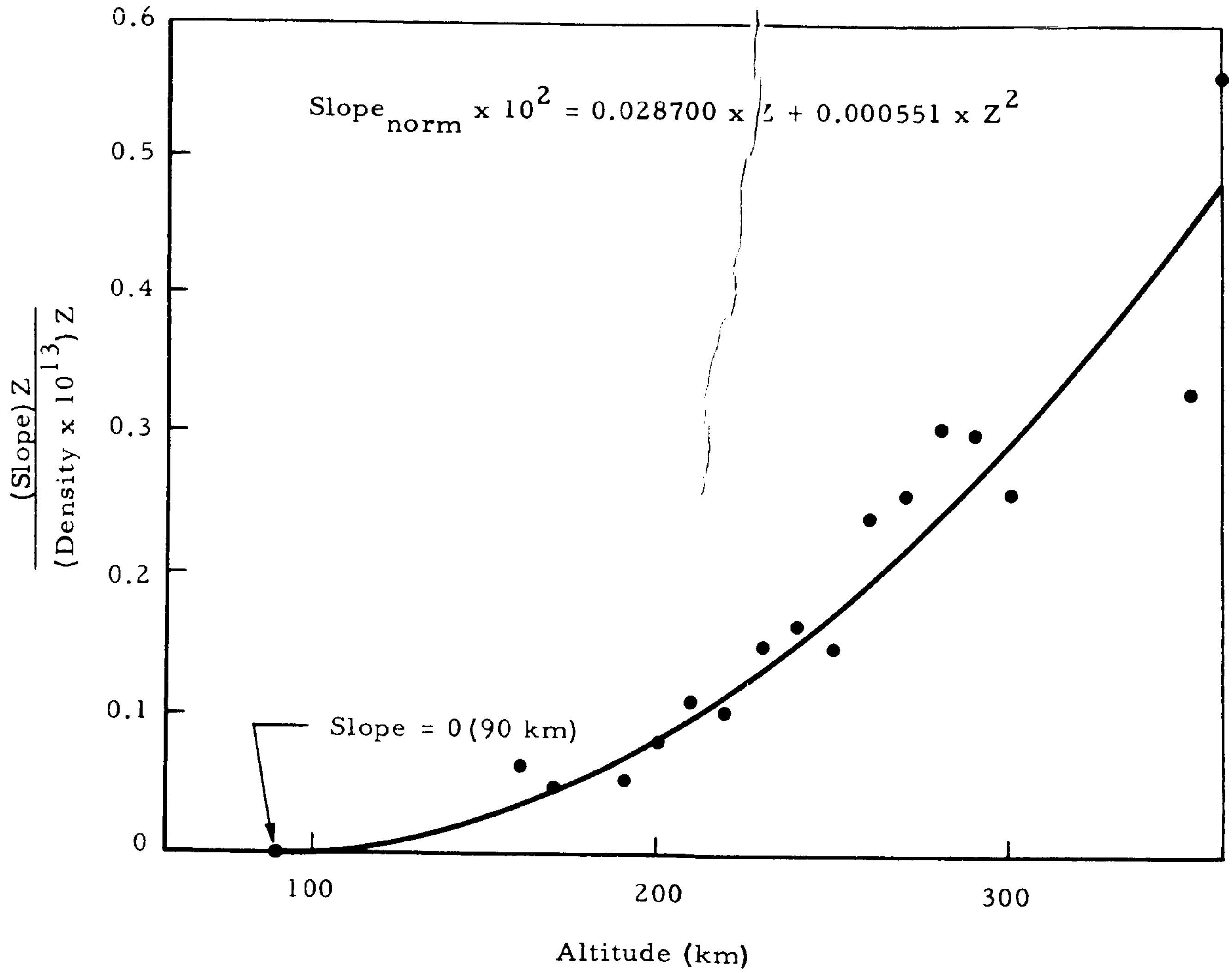

Fig. 2 - Normalized Density vs Altitude

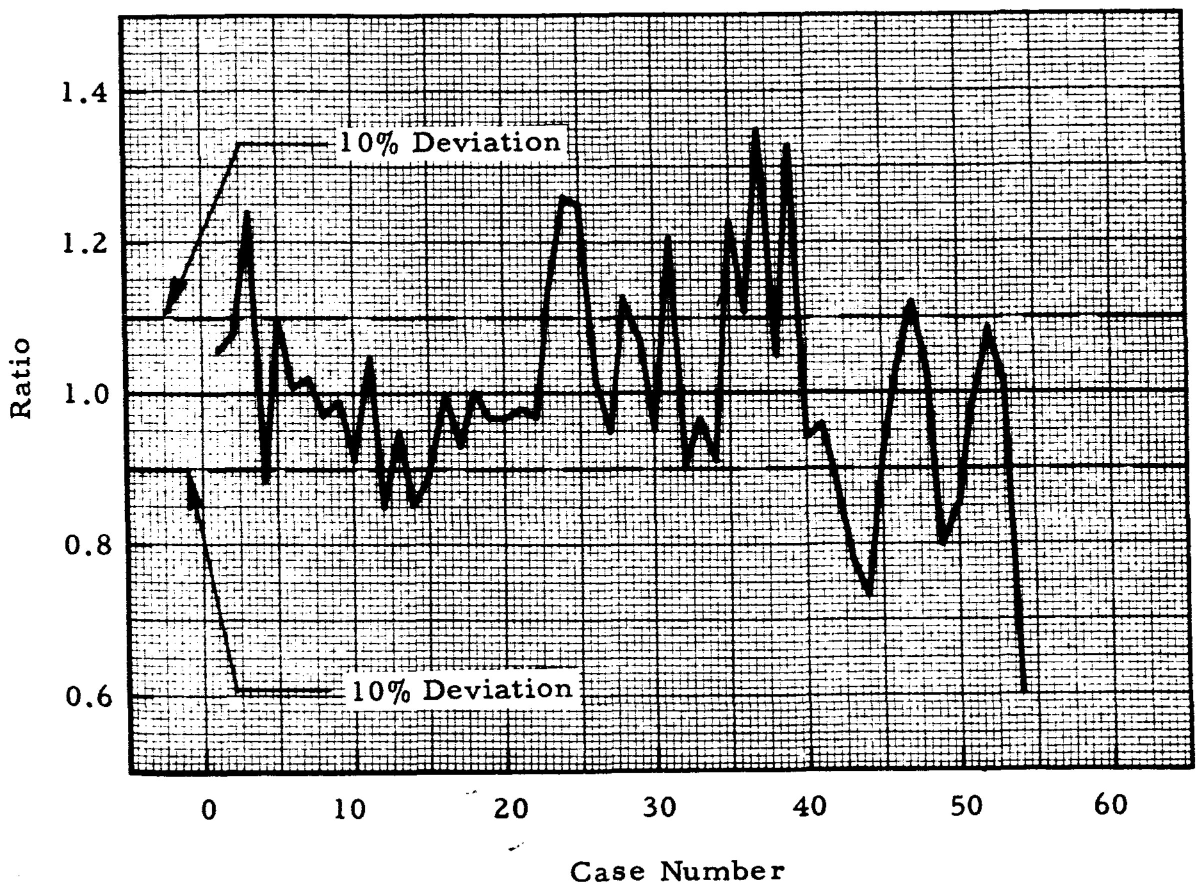

Fig. 3 - Ratios of Actual Lifetimes to Computed Lifetimes Using Linear Model

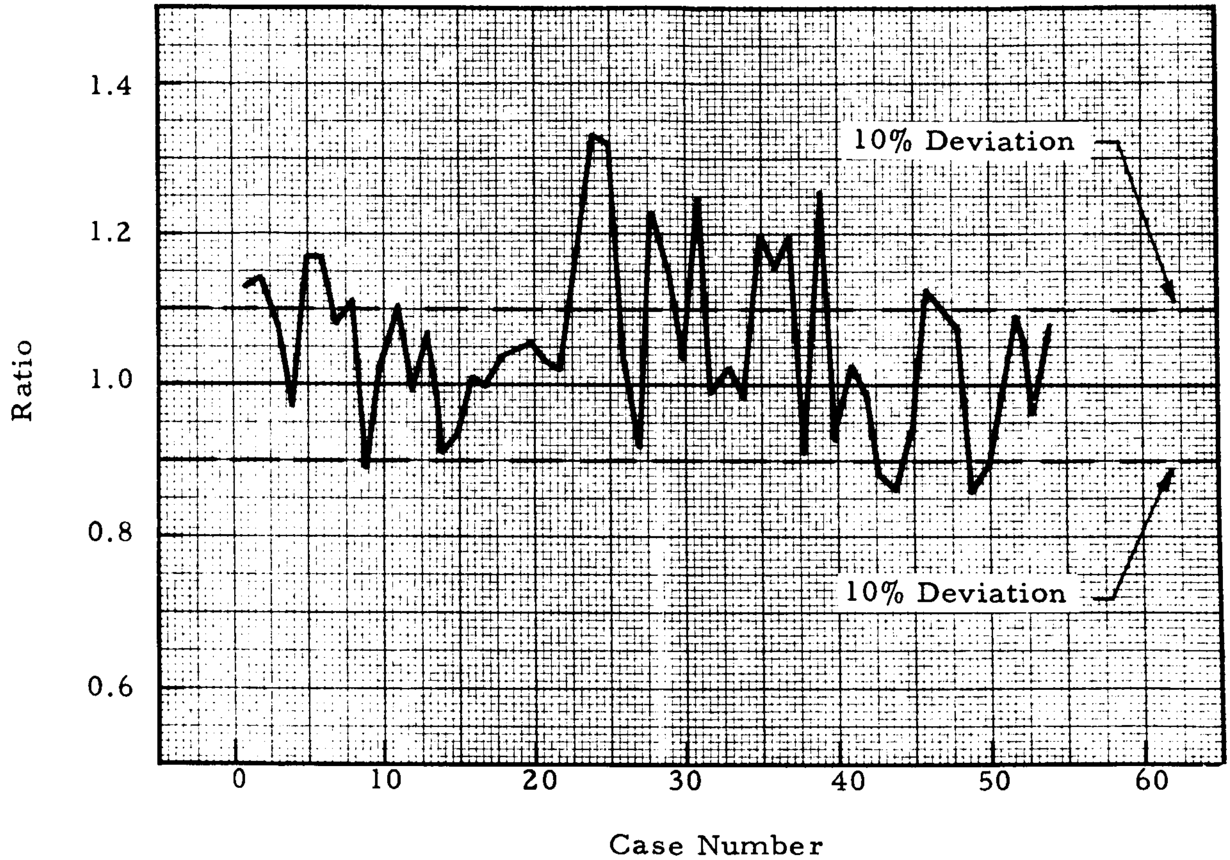

Fig. 4 - Ratios of Actual Lifetimes to Computed Lifetimes Using 1966 Jacchia Model

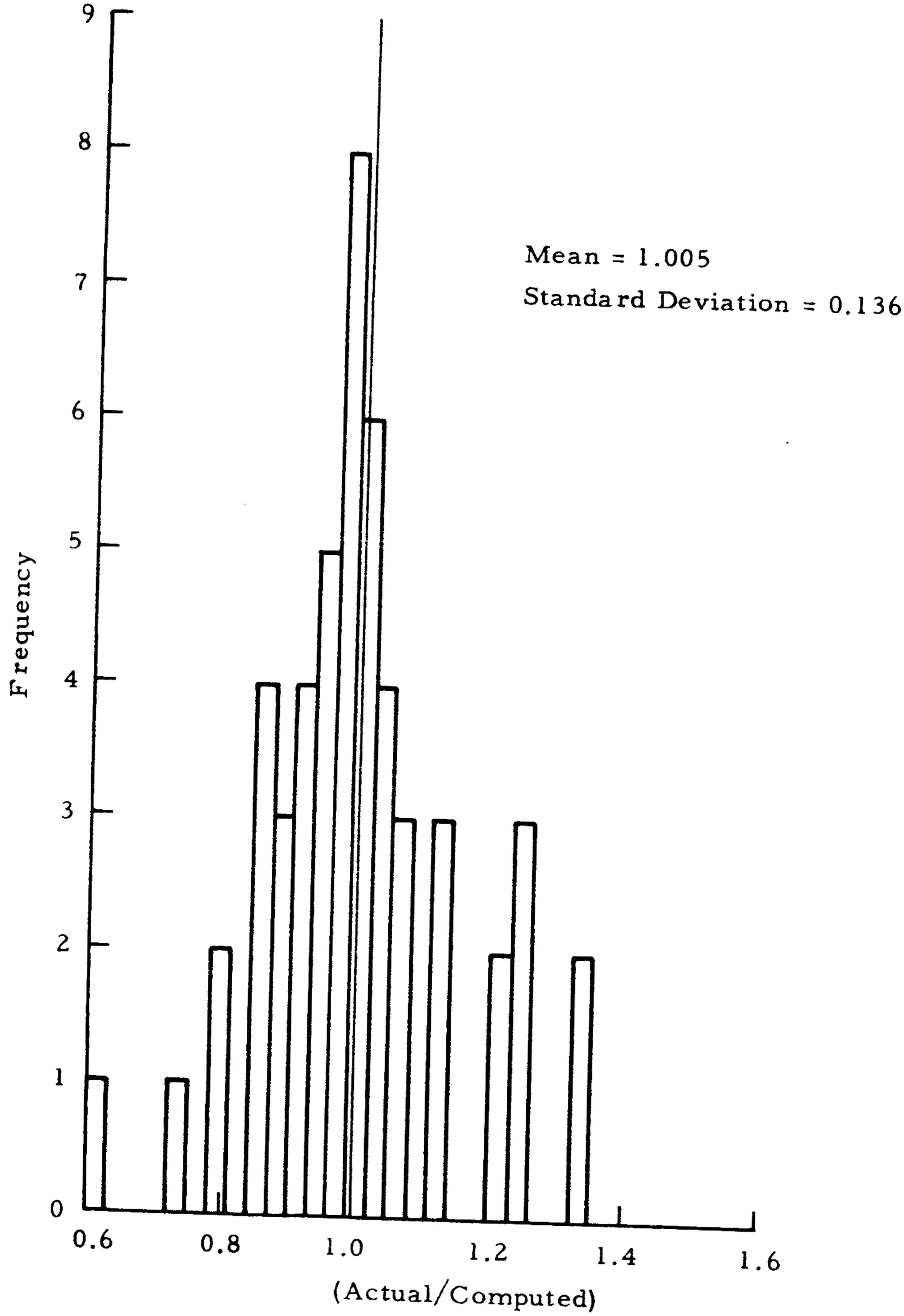

Fig. 5 - Histogram of Ratio of Actual to Computed Lifetimes Using Linear Model

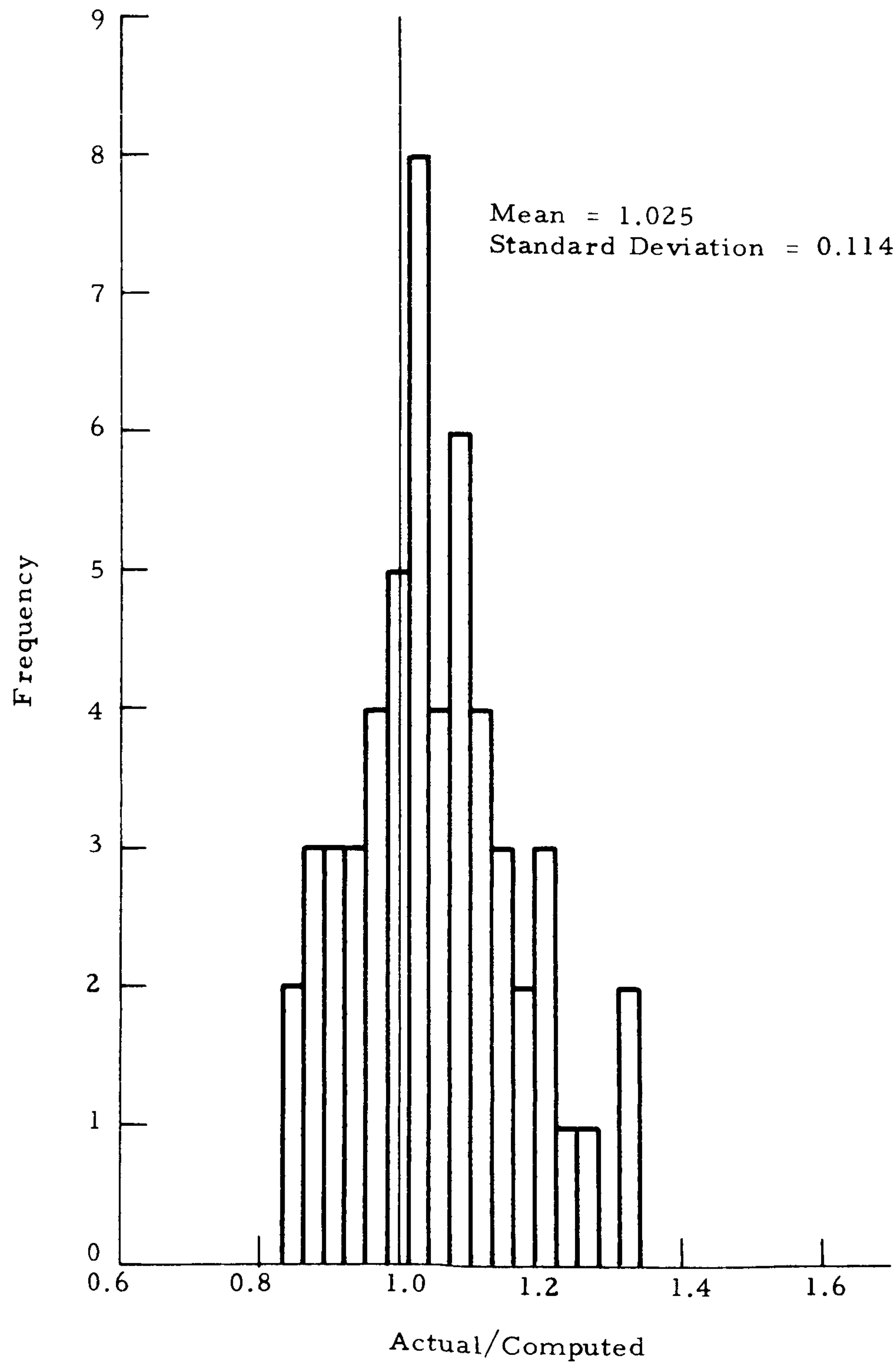

Fig. 6 - Histogram of Ratio of Actual to Computed Lifetimes Using 1966 Jacchia Model

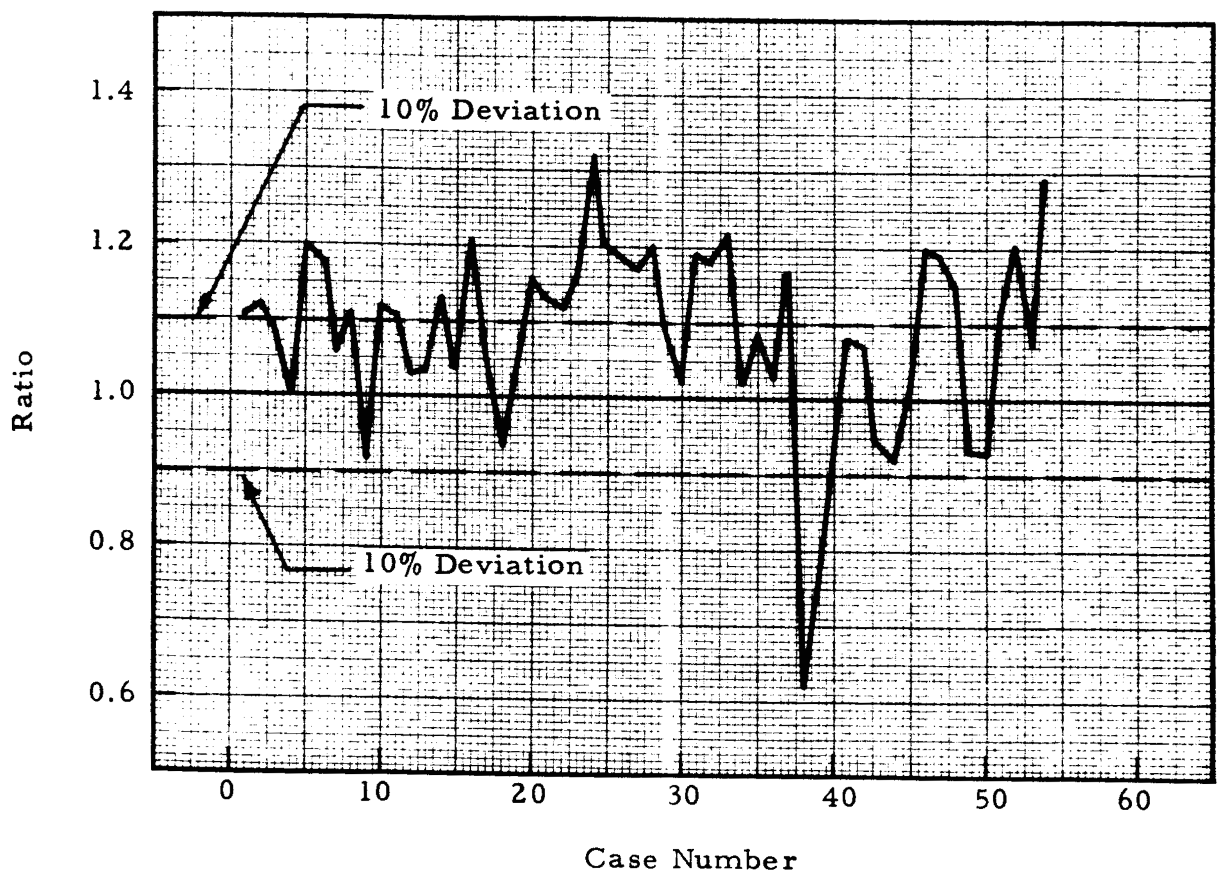

Fig. 7 - Ratios of Actual Lifetimes to Computed Lifetimes Using 1967 LMSC Model

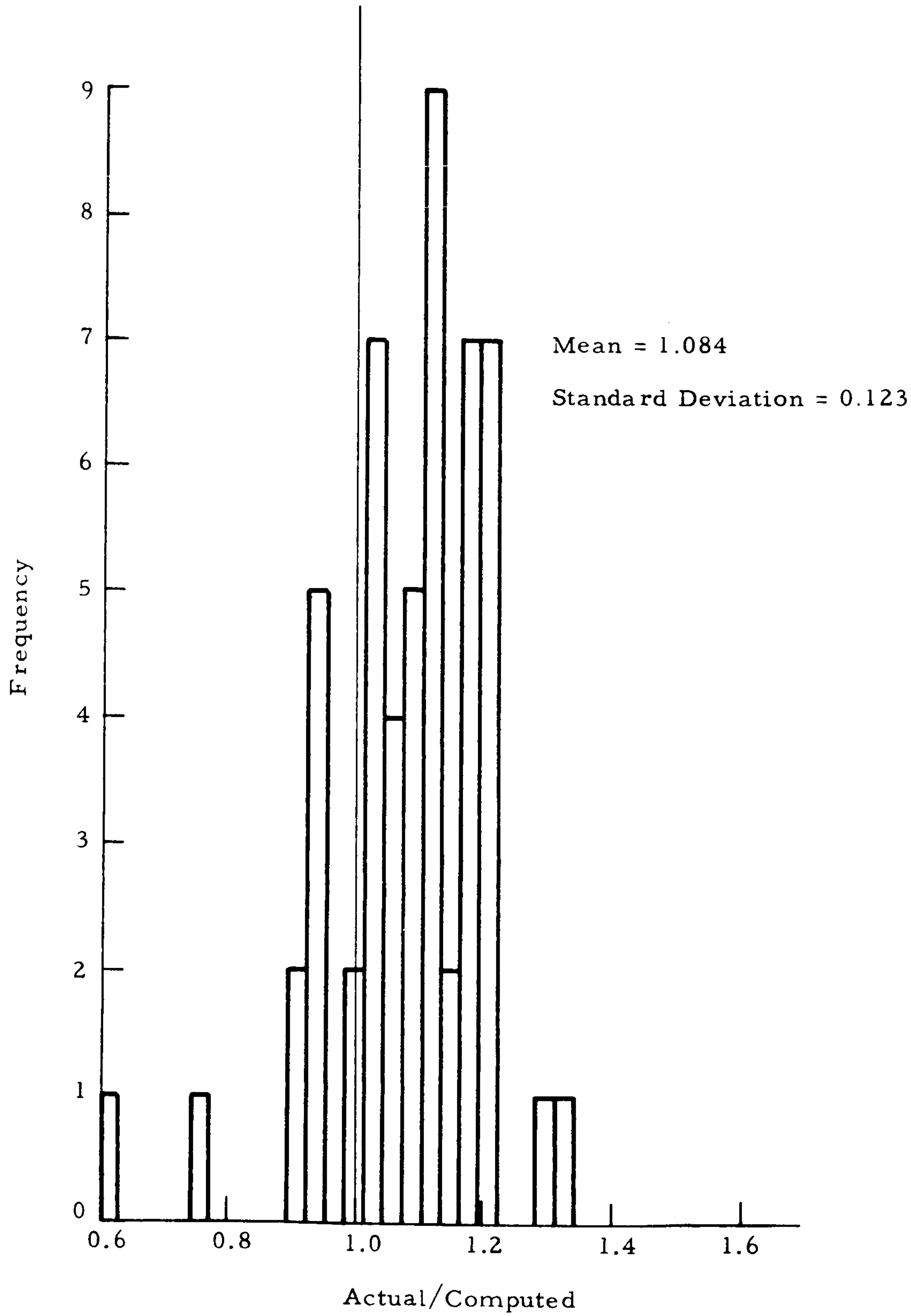

Fig. 8 - Histogram of Ratio of Actual to Computed Lifetimes Using 1967 LMSC Model

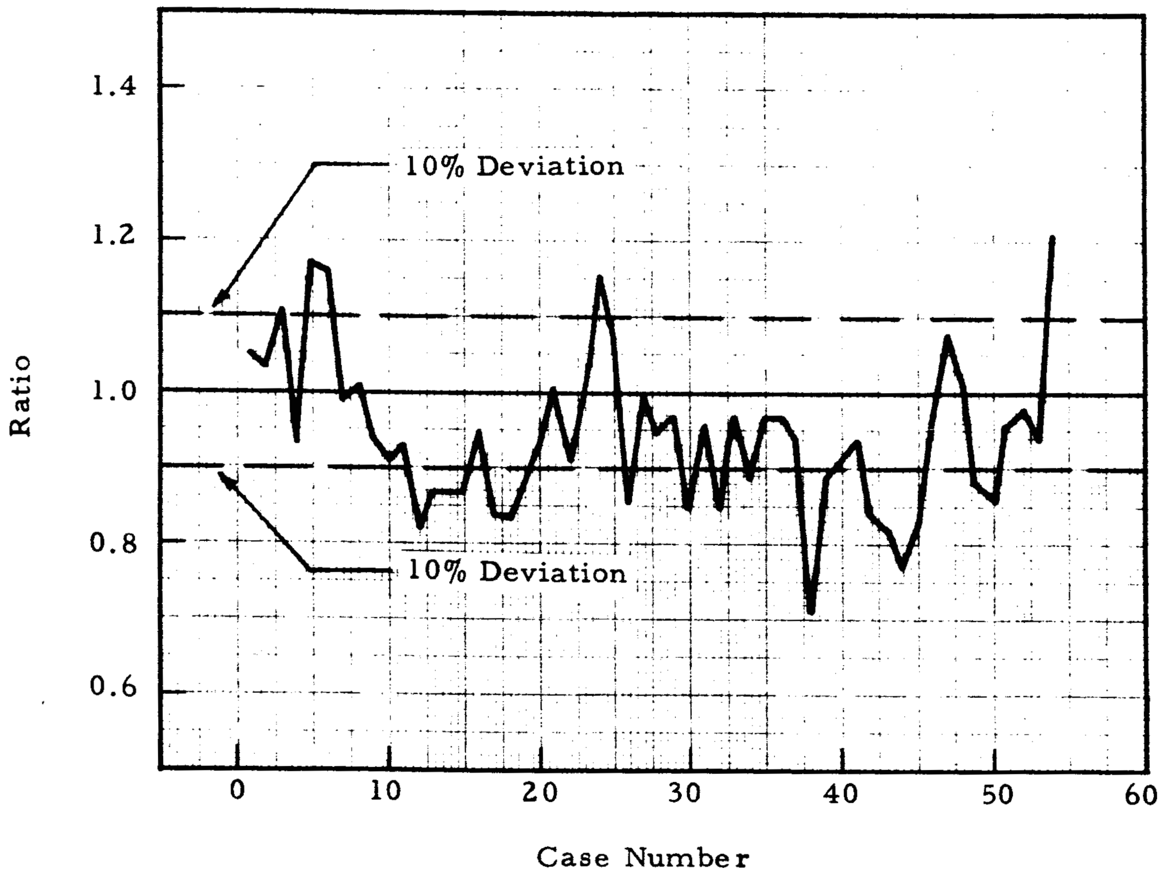

Fig. 9 - Ratios of Actual Lifetimes to Computed Lifetimes Using 1962 Special Model

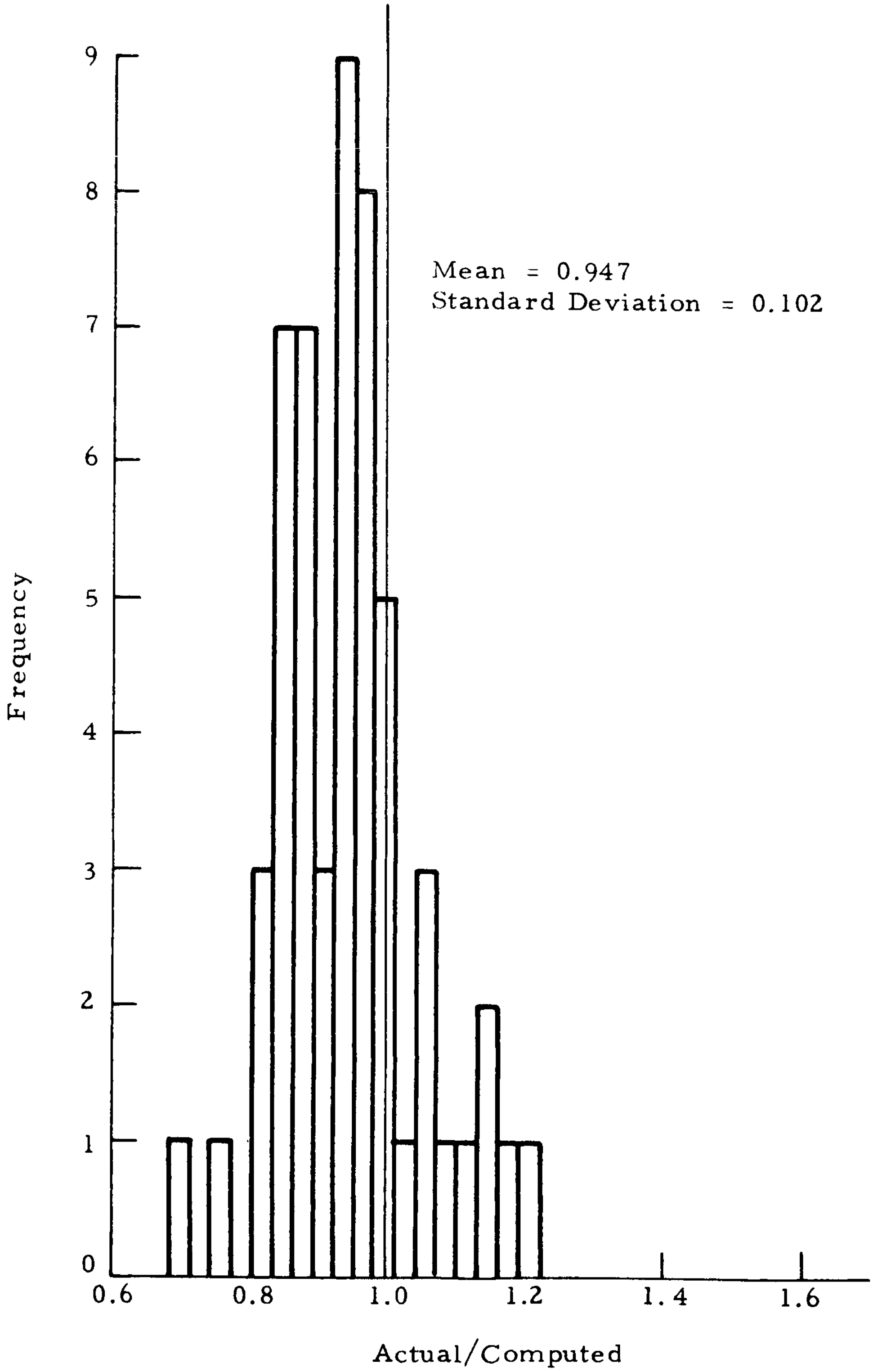

Fig. 10 - Histogram of Ratio of Actual to Computed Lifetimes Using 1962 Special Model

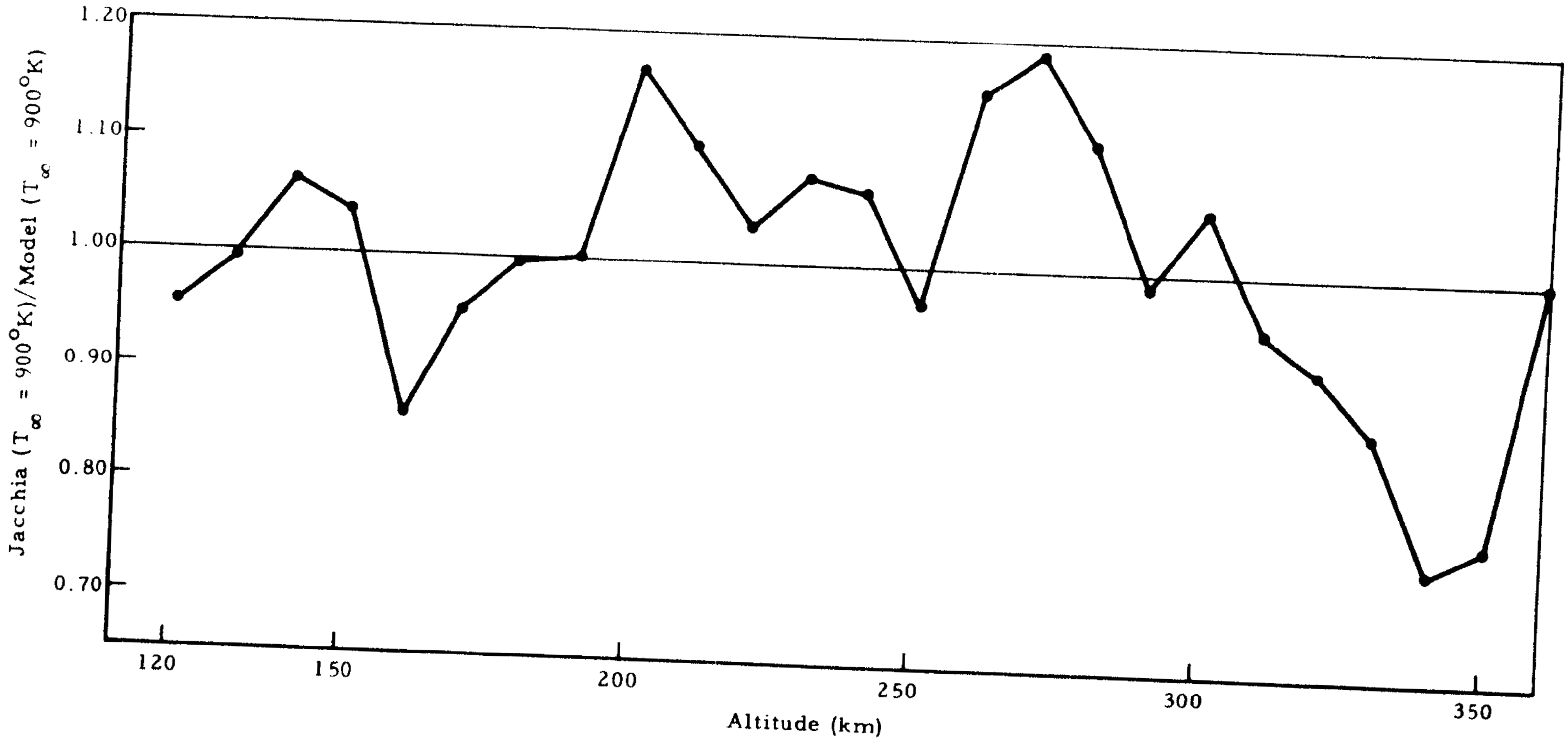

Fig. 11 - Ratio of Jacchia Density and Model Density (T_∞ = 900) vs Altitude